

Example 1 Determine the domain of each of the following.

(a) $f(x, y) = \sqrt{x+y}$

(b) $f(x, y) = \sqrt{x} + \sqrt{y}$

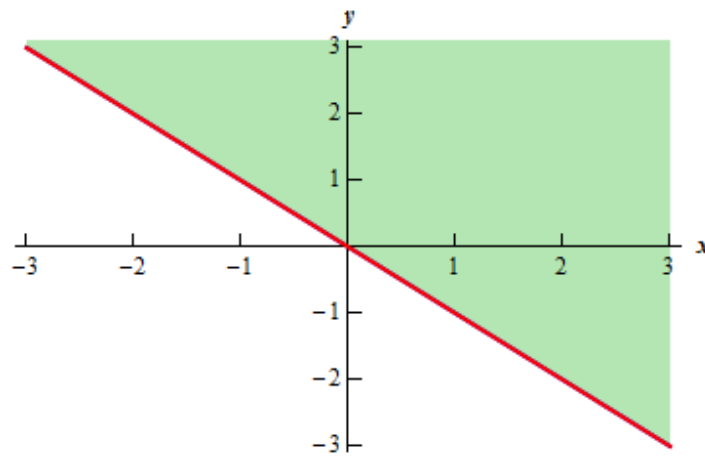
(c) $f(x, y) = \ln(9 - x^2 - 9y^2)$

Solution

(a) In this case we know that we can't take the square root of a negative number so this means that we must require,

$$x + y \geq 0$$

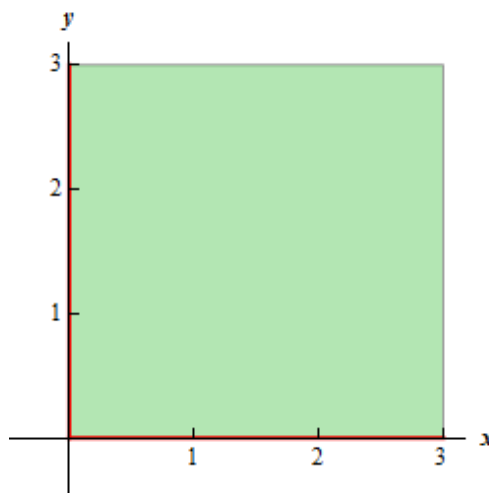
Here is a sketch of the graph of this region.



(b) This function is different from the function in the previous part. Here we must require that,

$$x \geq 0 \quad \text{and} \quad y \geq 0$$

and they really do need to be separate inequalities. There is one for each square root in the function. Here is the sketch of this region.



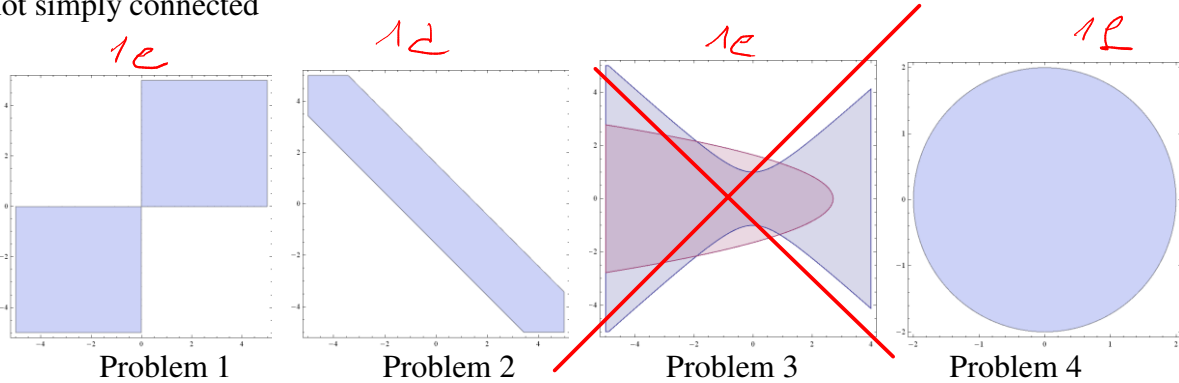
(c) In this final part we know that we can't take the logarithm of a negative number or zero. Therefore, we need to require that,

Functions of two variables

1e 1. Determine and sketch domain of definition of function $f(x, y) = \ln(xy)$. Describe properties of $D(f)$. Does point $A = [-1, 0]$ belong to $D(f)$? Is A a boundary point of $D(f)$?
Solution: $D(f) = \{[x, y] \in \mathbb{R}^2 : (x < 0 \wedge y < 0) \vee (x > 0 \wedge y > 0)\}$, $A \notin D(f)$, A is boundary point

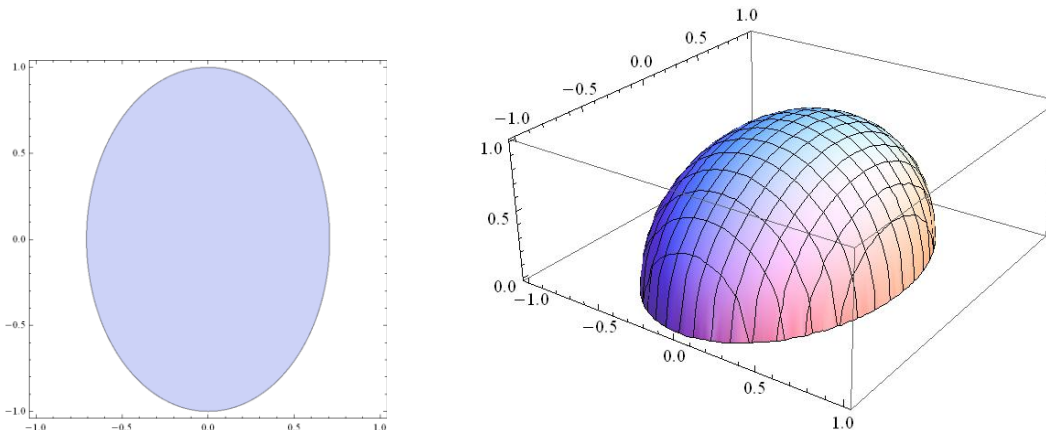
1e 2. Determine and sketch domain of definition of function $f(x, y) = 1 + \arcsin(x + y)$. Describe properties of $D(f)$. Is point $A = [0, 0]$ an interior point of $D(f)$?
Solution: $D(f) = \{[x, y] \in \mathbb{R}^2 : -\frac{\pi}{2} < x + y < \frac{\pi}{2}\}$, $A \in D(f)$, A is interior point

1e 3. Determine and sketch domain of definition of function $f(x, y) = \frac{1}{x^2 - y^2 + 1} + \ln(e - y - x^2)$. Describe its properties.
Solution: $D(f) = \{[x, y] \in \mathbb{R}^2 : x^2 - y^2 + 1 \neq 0 \wedge e - y - x^2 > 0\}$, $D(f)$ is open, not bounded, not simply connected

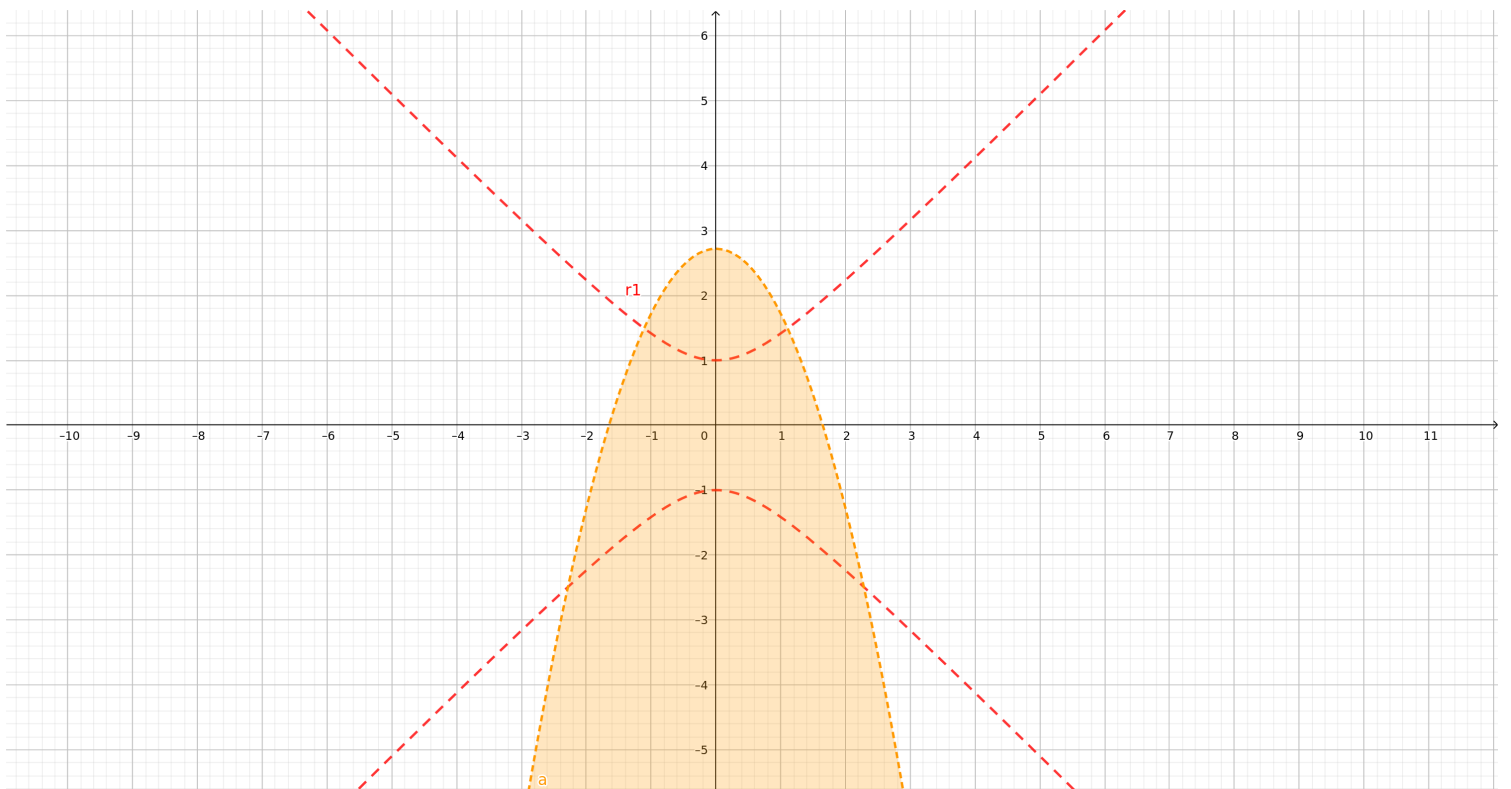


1e 4. Determine and sketch domain of definition of function $f(x, y) = 2 - \sqrt{4 - x^2 - y^2}$. Calculate $f(0, 1)$, $f(-1, 1)$. Describe graph of function f .
Solution: $D(f) = \{[x, y] \in \mathbb{R}^2 : 4 - x^2 - y^2 \geq 0\}$, $f(0, 1) = 2 - \sqrt{3}$, $f(-1, 1) = 2 - \sqrt{2}$, graph is semi-sphere with centre in point $S = [0, 0, 2]$ and radius 2.

1g 5. Determine and sketch domain of definition of function $f(x, y) = \sqrt{1 - 2x^2 - y^2}$. Sketch and describe graph of function f .
Solution: $D(f) = \{[x, y] \in \mathbb{R}^2 : 1 - 2x^2 - y^2 \geq 0\}$, graf is a semi-ellipsoid



1e

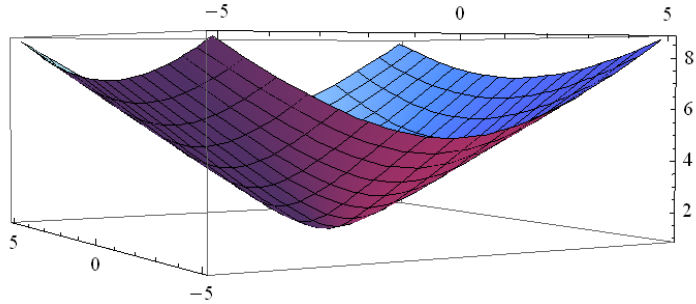
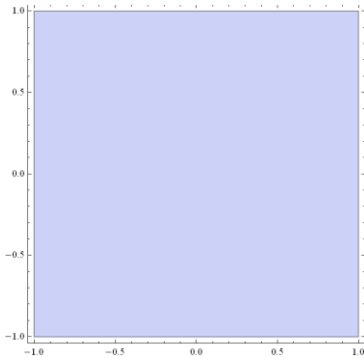


1e

6. Determine and sketch domain of definition of function $f(x, y) = \sqrt{2x^2 + y^2 + 1}$.

Sketch and describe graph of function f .

Solution: $D(f) = \{[x, y] \in \mathbb{R}^2 : 2x^2 + y^2 + 1 \geq 0\}$, graph is a half-conical surface

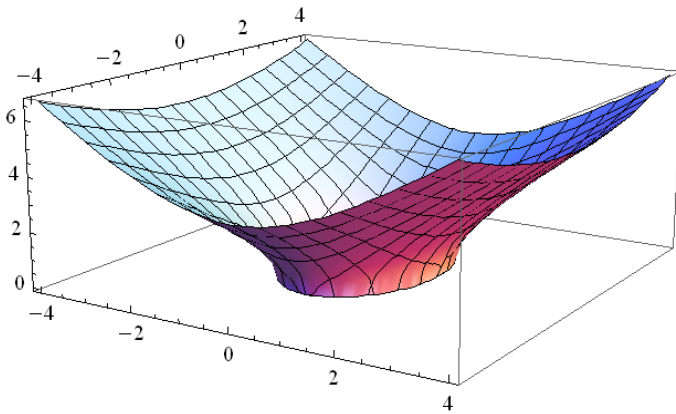
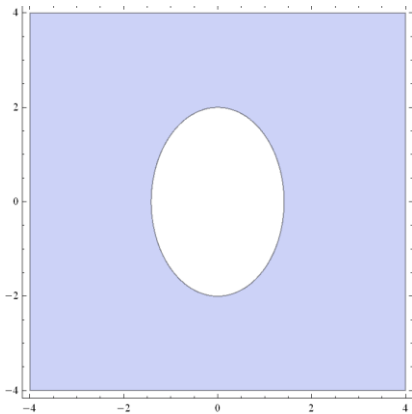


1i

7. Determine and sketch domain of definition of function $f(x, y) = \sqrt{2x^2 + y^2 - 4}$

Sketch and describe graph of function f .

Solution: $D(f) = \{[x, y] \in \mathbb{R}^2 : 2x^2 + y^2 - 4 \geq 0\}$, graph is semi.hyperboloid

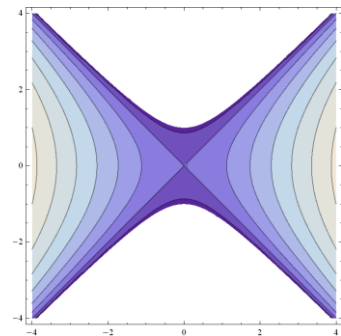
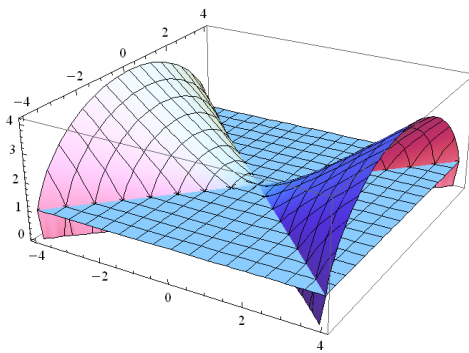
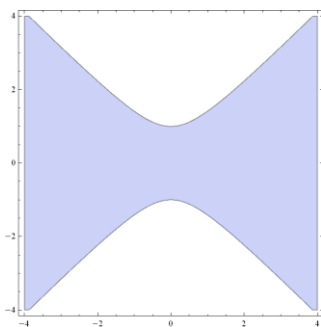


1j

8. Determine and sketch domain of definition of function $f(x, y) = \sqrt{x^2 - y^2 + 1}$

Sketch graph of function f , its level lines and find planar intersection by plane $z = 1$.

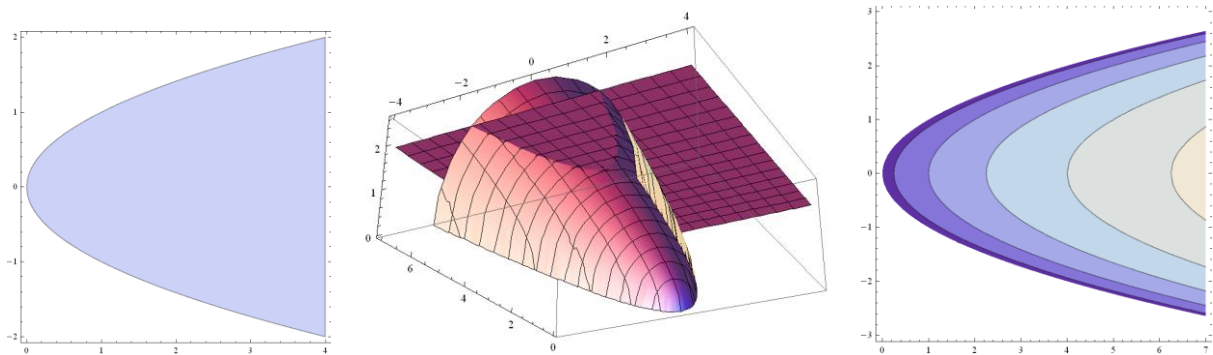
Solution: $D(f) = \{[x, y] \in \mathbb{R}^2 : x^2 - y^2 + 1 \geq 0\}$



9. Determine and sketch domain of definition of function $f(x, y) = \sqrt{x - y^2}$.

Sketch graph of function f , its level lines and find planar intersection by plane $z = 2$.

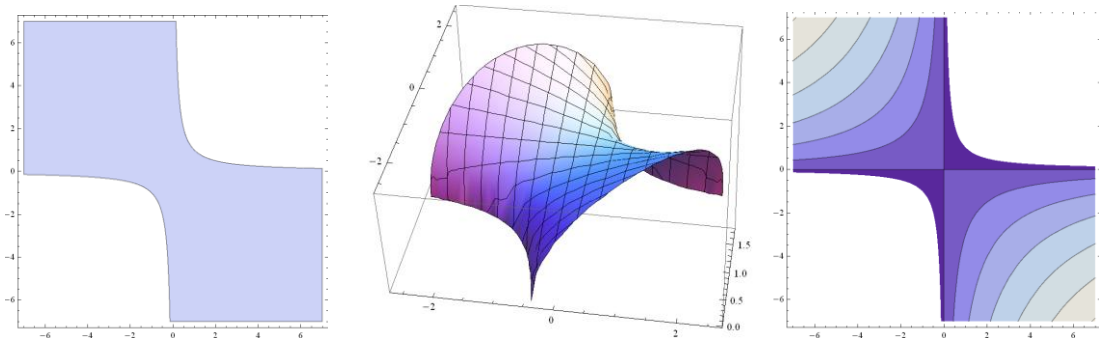
Solution: $D(f) = \{[x, y] \in \mathbb{R}^2 : x - y^2 \geq 0\}$



10. Determine and sketch domain of definition of function $f(x, y) = \sqrt{1 - xy}$.

Nakreslite graf danej funkcije. Nakreslite vrstevnice grafu.

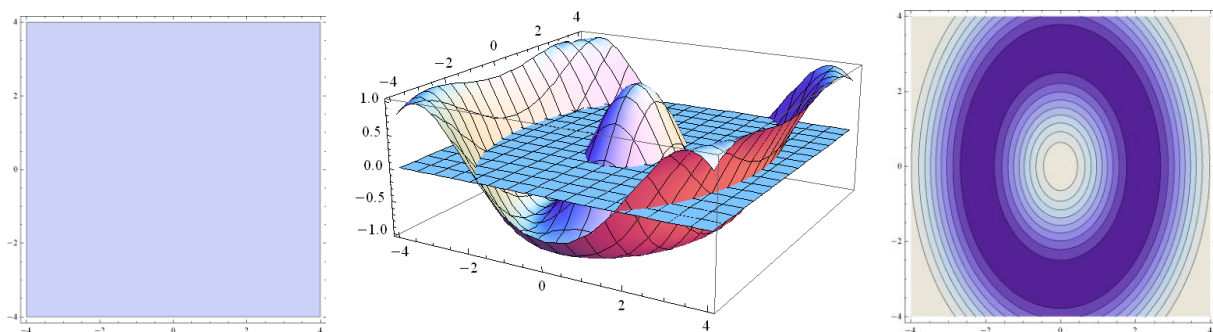
Solution: $D(f) = \{[x, y] \in \mathbb{R}^2 : 1 - xy \geq 0\}$



11. Determine and sketch domain of definition of function $f(x, y) = \cos\sqrt{2x^2 + y^2}$.

Sketch graph of function f , its level lines and find planar intersection by plane $z = 0$.

Solution: $D(f) = \{[x, y] \in \mathbb{R}^2 : 2x^2 + y^2 \geq 0\}$



8th lesson

<https://www2.karlin.mff.cuni.cz/~kuncova/en/teaching.php>
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Theory

Exercises

1. Find and sketch the domain:

(a) $f(x, y) = \sqrt{x + y}$

(g) $f(x, y) = \sqrt{1 - 2x^2 - y^2}$

(b) $f(x, y) = \sqrt{x} + \sqrt{y}$

(h) $f(x, y) = \sqrt{2x^2 + y^2 + 1}$

(c) $f(x, y) = \ln(xy)$

(i) $f(x, y) = \sqrt{2x^2 + y^2 - 4}$

(d) $f(x, y) = 1 + \arcsin(x + y)$

(e) $f(x, y) = \frac{1}{x^2 - y^2 + 1} + \ln(e - y - x^2)$

(j) $f(x, y) = \sqrt{x^2 - y^2 + 1}$

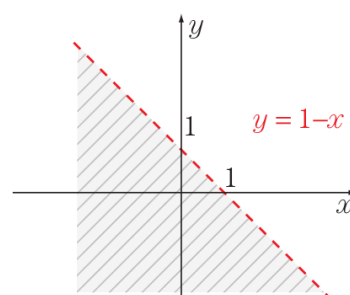
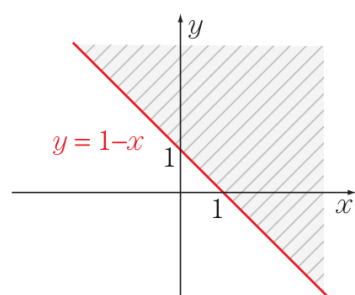
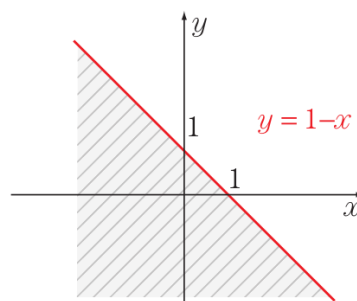
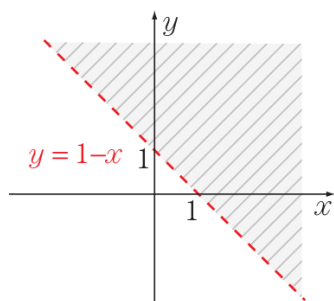
(f) $f(x, y) = 2 - \sqrt{4 - x^2 - y^2}$

(k) $f(x, y) = \sqrt{1 - xy}$

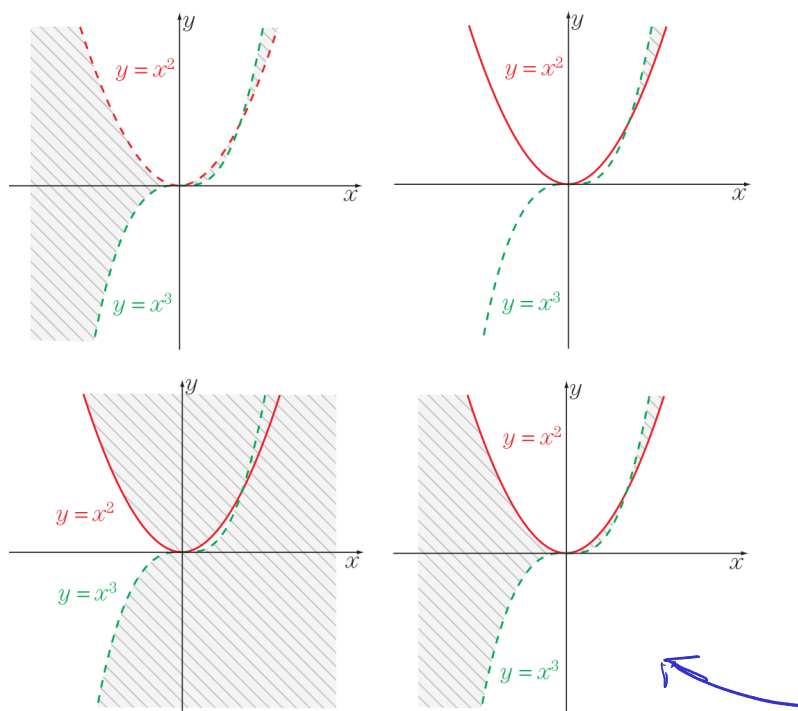
The following exercise is by http://homel.vsb.cz/~kre40/esfmat2/kapitoly/kapitola_4.1.pdf

2. Find the domain

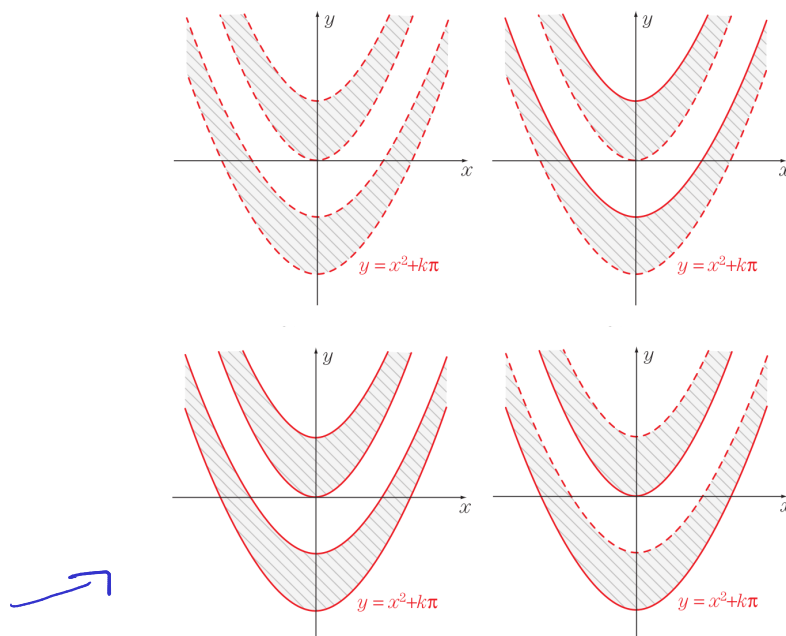
(a) $f(x, y) = \sqrt{\ln(x + y)}$



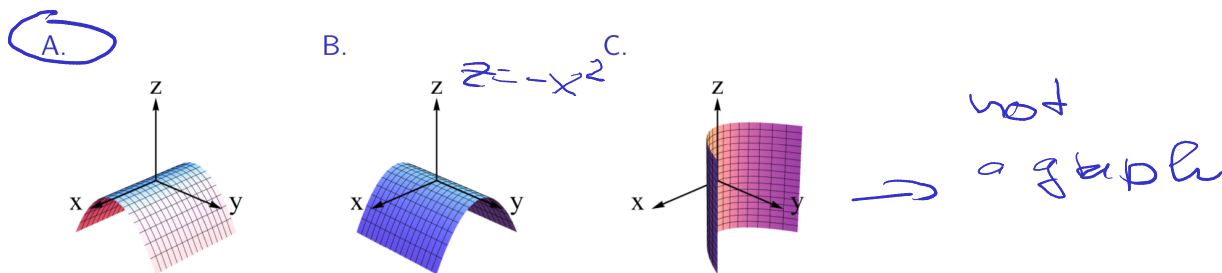
$$(b) f(x, y) = \sqrt{\frac{y-x^2}{x^3-y}}$$



$$(c) f(x, y) = \sqrt{\sin(y - x^2)}$$



3. Find the graph of function $z = -y^2$:



Source 1: <http://www.cpp.edu/~conceptests/question-library/mat214.shtml>

4. Which of the following objects can NOT be a graph of two variable function:

- (a) plane (board);
- (b) cylinder without base (pipe);
- (c) sphere (orange peel);
- (d) paraboloid (Davis cup);
- (e) line.