

$$\begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix} = 1 \cdot 2 - 2 \cdot 3 = -2$$

$$\begin{pmatrix} 2 & -3 \\ -6 & 9 \end{pmatrix} \sim \begin{pmatrix} 2 & -3 \\ 0 & 0 \end{pmatrix}$$

↑  
bad guy without inverse

$$A^{-1} \text{ exists} \iff \det A \neq 0$$

$$\left( \begin{array}{ccc|cc} 1 & 0 & 2 & 1 & 0 \\ -3 & 1 & 4 & -3 & 1 \\ 0 & 2 & 5 & 0 & 2 \end{array} \right)$$

$$\begin{aligned} & 1 \cdot 1 \cdot 5 + 0 \cdot 4 \cdot 0 + 2 \cdot (-3) \cdot 2 \\ & - (0 \cdot 1 \cdot 2 + 2 \cdot 4 \cdot 1 + 5 \cdot (-3) \cdot 0) \\ & = 5 - 12 - (8 - 15) = \\ & = -7 - (-7) = 0 \end{aligned}$$

$$\begin{vmatrix} 2 & 3 & 1 \\ 0 & 5 & -2 \\ 0 & 0 & -2 \end{vmatrix} = -20$$

S<sub>2</sub> ↙

$$\begin{vmatrix} 1 & 0 & 2 & 3 \\ 2 & 1 & 0 & -1 \\ 3 & 2 & 0 & 4 \\ 0 & 1 & 2 & -2 \end{vmatrix}$$

$$= (-1)^{1+3} \cdot 2 \cdot \begin{vmatrix} 2 & 1 & -1 \\ 3 & 2 & 4 \\ 0 & 1 & -2 \end{vmatrix}$$

$$+ (-1)^{2+3} \cdot 0 \cdot \begin{vmatrix} 1 & 0 & 3 \\ 3 & 2 & 4 \\ 0 & 1 & -2 \end{vmatrix}$$

$$+ (-1)^{3+3} \cdot 0 \cdot \begin{vmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 0 & 1 & -2 \end{vmatrix}$$

$$+ (-1)^{4+3} \cdot 2 \cdot \begin{vmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 3 & 2 & 4 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 0 & 3 \\ 3 & 2 & 4 \\ 0 & 1 & -2 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 0 & 1 & -2 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 3 & 2 & 4 \end{vmatrix}$$

$$= 2 \cdot (-8 - 3 - (8 - 6)) + 0 + 0$$

$$-1 \cdot 2 \cdot (4 + 12 - (9 - 2)) =$$

$$= 2 \cdot (-11 - 2) - 2 \cdot 9 = -26 - 18 = -44$$