

Rank

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 3 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{rank} = 3$$

$$\begin{matrix} \text{+} \\ \text{+} \end{matrix} \begin{pmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 0 & 3 & 5 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 3 & 5 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & 0 \end{pmatrix}$$

rank = 2

$$\text{rank } A = \text{rank } A^T$$

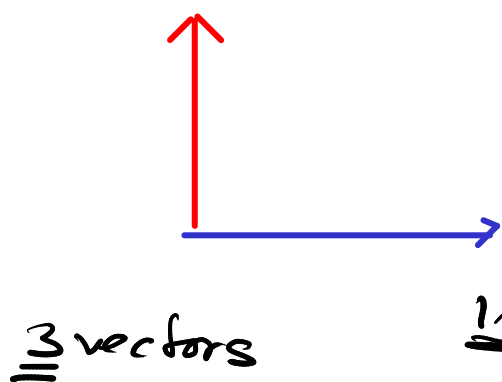
$$\begin{vmatrix} 1 & 0 & 2 \\ -1 & 2 & 0 \\ 0 & 2 & 2 \end{vmatrix} = 4 - 4 = 0 \quad \text{rank} = 2$$

$$\begin{matrix} \text{+} \\ \text{+} \end{matrix} \begin{pmatrix} 1 & 0 & 2 \\ -1 & 2 & 0 \\ 0 & 2 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 2 \\ 0 & 2 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

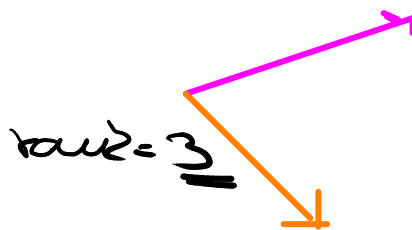
$$\begin{vmatrix} 1 & 0 & 0 \\ 2 & 0 & 0 \\ 3 & 0 & 0 \end{vmatrix} = 0 \quad \begin{matrix} \text{+} \\ \text{+} \end{matrix} \begin{pmatrix} 1 & 0 & 0 \\ 2 & 0 & 0 \\ 3 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\text{rank} = 1$$

$$\det \neq 0 \\ \text{rank} = 3$$



NO



$$\begin{pmatrix} -1 & 3 \\ 5 & 6 \\ 1 & 4 \end{pmatrix}$$

rank = 2

Dependent

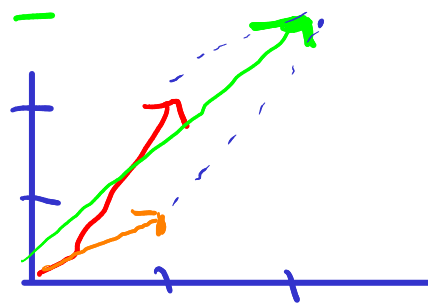
$(2, 3)$  express using

$(1, 2)$  and  $(1, 1)$

$$(2, 3) = a(1, 2) + b(1, 1) \quad ?$$

$$2 = a + b$$

$$3 = 2a + b$$



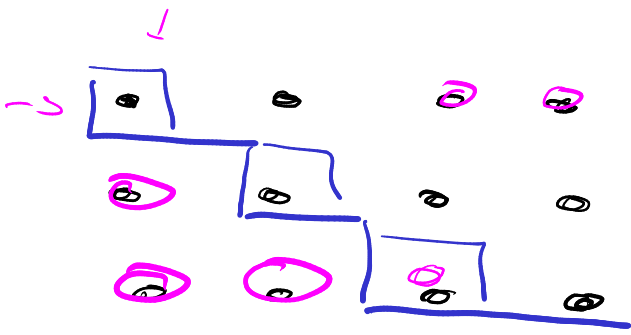
$$\boxed{1 = a}$$

$$2 = 1 + b$$

$$\rightarrow \boxed{b = 1}$$

$$\underline{\underline{(2, 3) = (1, 2) + (1, 1)}}$$

$$(0,0) = \underset{0}{\overset{1}{\cdot}}(1,2) + \underset{0}{\overset{1}{\cdot}}(1,1) - \underset{0}{\overset{1}{\cdot}}(2,3)$$



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