Definiteness — matrix A symmetric $n \times n$ $A = A^T$

pos. definite

neg definite

indefinite

$\begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$ ...

$\begin{pmatrix} 3 & 0 & 2 \\ 0 & 1 & 7 \\ 2 & 7 & 5 \end{pmatrix}$

$\begin{matrix} 1 & 2 \\ 1 & 5 \end{matrix}$ $\times$

Eigenvalues

A

$\begin{pmatrix} 5 & -2 \\ -2 & 5 \end{pmatrix}$

$\begin{vmatrix} 5-\lambda & -2 \\ -2 & 5-\lambda \end{vmatrix} = (5-\lambda)(5-\lambda) - (-2)(-2)$ → diagonal

$= 25 + \lambda^2 - 10\lambda - 4 =$

$= \lambda^2 - 10\lambda + 21$

$\lambda^2 - 10\lambda + 21 = 0$

$(\lambda - 7)(\lambda - 3) = 0$

eigenvalues

$\lambda_1 = 7 > 0$

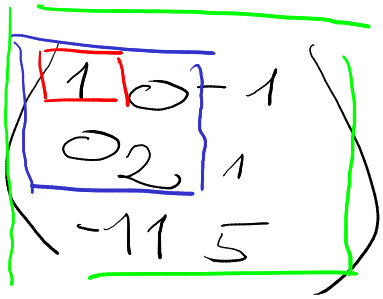
$\lambda_2 = 3 > 0$

7: $\begin{vmatrix} -2 & -2 \\ -2 & -2 \end{vmatrix} = 0$

3: $\begin{vmatrix} 2 & -2 \\ -2 & 2 \end{vmatrix} = 0$

By the eginvalue test

A is positive definite



$D_1 = 1 > 0$

$D_2 = 1 \cdot 2 - 0 \cdot 0 = 2 > 0$

$D_3 = 10 - 2 - 1 = 7 > 0$

$D_1, D_2, D_3 > 0$ matrix positive definite