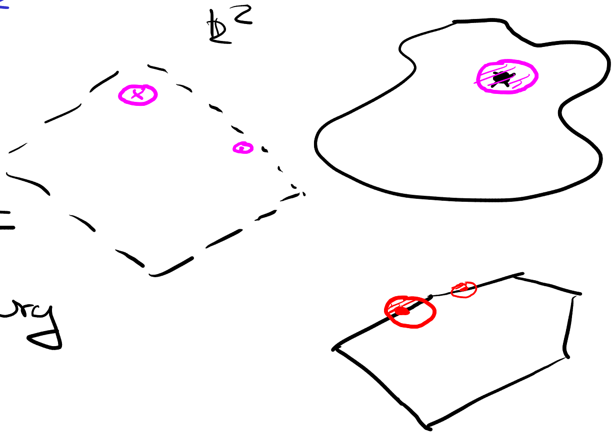
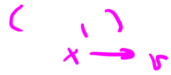
Open set $\mathbb{R}^2$ $\mathbb{R}^1$ has not boundary

Ball



$\mathbb{R}^1$



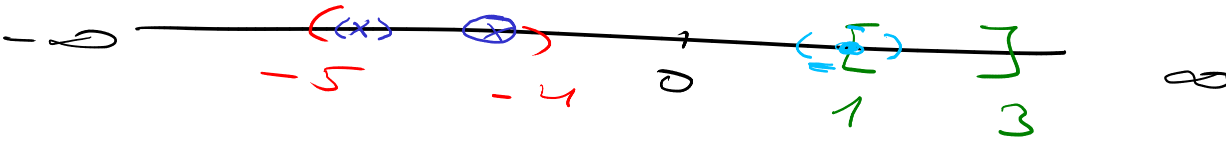
$B(2, \frac{1}{2})$

Closed set F

has boundary

$\mathbb{R}^n \setminus F$ is open

$\mathbb{R}^1$



open   not open

— Open set $G_1 \cup G_2$ open

—

$(-\infty, \infty)$ open

$(-\infty, 1)$ open $(-\infty, 1)$

$(3, \infty)$ open $\cup (3, \infty)$ open :)

$[1, 3]$ closed ? ✓

$$\mathbb{R} \setminus [1,3] = \underbrace{(-\infty, 1) \cup (3, \infty)}_{\text{open}}$$

— $\mathbb{R}, \emptyset$ open, closed "clopen"

Empty set → no elements

$[1, 2)$ $(-\infty, 1) \cup [2, \infty)$

open ?

no open

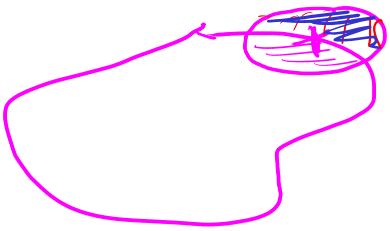
no closed :-(

$(-\infty, 2]$ not open

complement $(2, \infty)$ open

→ closed

Boundary

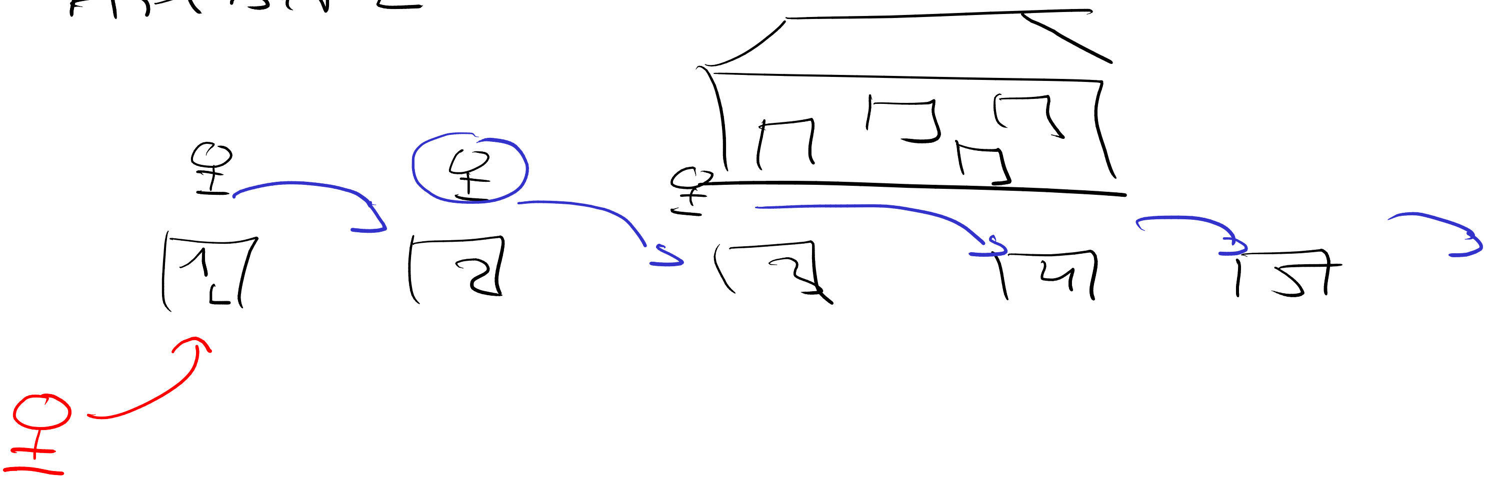


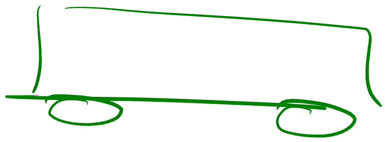
Interior

$\mathbb{N}$ 1   2   3   4 ∞

$N+1$ 0   1   2   3   4 $1+\infty$

Hilbert





-1 -2 -3

-4

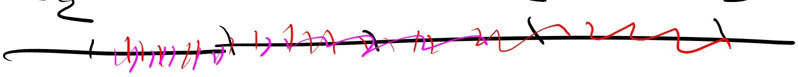
$1 \to 2$

$2 \to 4$

$3 - 6$

odd rooms :) free

$\mathbb{Q}$ can be in hotel



countable

$\mathbb{R} \setminus \mathbb{Q}$:(much more of them

uncountable