Tangent plane

- $\mathbb{R}^1$ $\quad y = ax + b$ line

f, x_0

$$y = f(x_0) + \underline{f'(x_0)}(x - x_0)$$

$$y = f(a) + f'(a)(x - \underline{a})$$

- $\mathbb{R}^2$ $\quad z = ax + by + c$ plane

$f \quad (x_0, y_0)$

$$z = f(x_0, y_0) + \frac{\partial f}{\partial x}(x_0, y_0)(x - x_0) + \frac{\partial f}{\partial y}(x_0, y_0)(y - y_0)$$

- $\mathbb{R}^3$ $\quad w = ax + by + cz + d$

$f \quad (x_0, y_0, z_0)$

$$w = f(x_0, y_0, z_0) + \frac{\partial f}{\partial x}(x_0, y_0, z_0)(x - x_0) + \frac{\partial f}{\partial y}(x_0, y_0, z_0)(y - y_0) + \frac{\partial f}{\partial z}(x_0, y_0, z_0) \cdot (z - z_0)$$

$$f(x,y) = x^2 - xy + 3y^3 \qquad A = [2, -1]$$

$$f(2,-1) = 3$$

$$\frac{\partial f}{\partial x} = 2x - 1y$$

$$\frac{\partial f}{\partial x}(2,-1) = 4 + 1 = 5$$

$$\frac{\partial f}{\partial y} = -x + 9y^2$$

$$\frac{\partial f}{\partial y}(2,-1) = -2 + 9 = 7$$

$$z = 3 + 5(x - 2) + 7(y - (-1))$$

$$z = 3 + 5x - 10 + 7y + 7$$

$$z = 5x + 7y$$

Partial derivative

$$\frac{\partial f}{\partial x} = 2xy - 1$$

at $(1,2) = 2\cdot 1\cdot 2 - 1 = 3$

$$f(x,y) = \underset{\uparrow\,1+t}{x}^2 \overset{2}{y} - \underset{\uparrow\,1+t}{x} \qquad [x,y] \in \mathbb{R}^2$$

$$\frac{\partial f}{\partial x}(x_0,y_0) = \lim_{t\to 0} \frac{f(x_0+\underline{t}, y_0) - f(x_0,y_0)}{\underline{t}}$$

$$\frac{\partial f}{\partial x}(1,2) = \lim_{t\to 0} \frac{f(1+t, 2) - f(1,2)}{t}$$

$$= \lim_{t\to 0} \frac{(1+t)^2\cdot 2 - (1+t) - (1^2\cdot 2 - 1)}{t}$$

$$= \lim_{t\to 0} \frac{(1+2t+t^2)\cdot 2 - 1 - t - (2-1)}{t}$$

$$= \lim_{t\to 0} \frac{2t^2 + \underline{2,t} + \cancel{2} - \cancel{1} \underline{-t} - \cancel{2} + \cancel{1}}{t}$$

$$= \lim_{t\to 0} \frac{2t^2 + 3t}{t} = \lim_{t\to 0} 2t+3 = \underline{\underline{3}}$$

$$f(x,y) = x^2 y - x$$

$$\frac{\partial f}{\partial y} = x^2 \quad \text{at } (x_0, y_0) = x_0^2$$

$$\frac{\partial f}{\partial y}(x_0, y_0) = \lim_{t \to 0} \frac{f(x_0, y_0 + t) - f(x_0, y_0)}{t}$$

$$= \lim_{t \to 0} \frac{\left(x_0^2 (y_0 + t) - x_0\right) - (x_0^2 y_0 - x_0)}{t}$$

$$= \lim_{t \to 0} \frac{\cancel{x_0^2 y_0} + t x_0^2 - \cancel{x_0} - \cancel{x_0^2 y_0} + \cancel{x_0}}{t}$$

$$= \lim_{t \to 0} x_0^2 = \underline{\underline{x_0^2}} \checkmark$$