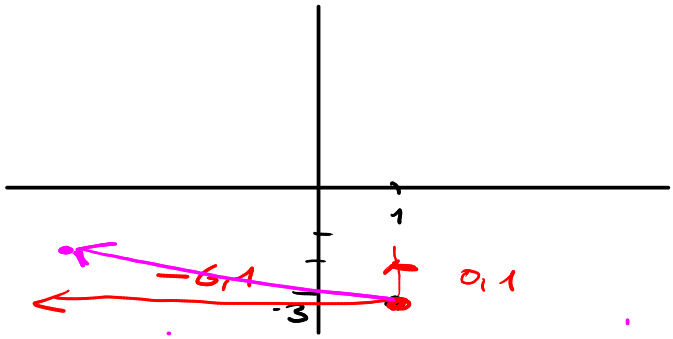
$f(x,y)$

Gradient

$$\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right)$$



$$f(x,y) = x^2y - 2xe^y$$

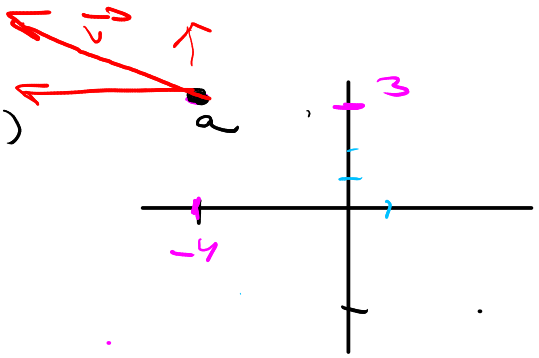
$$\nabla f = (2xy - 2e^y, x^2 - 2xe^y)$$

$$\nabla f(1,-3) = (2\cdot 1(-3) - 2e^{-3}; 1^2 - 2\cdot 1\cdot e^{-3}) =$$

$$= (\underbrace{-6 - 2e^{-3}}_{-6,1}, \underbrace{1 - 2e^{-3}}_{0,9})$$

Directional derivative

$$f = x^2 + y^2 \qquad \vec{v} = (-4,3) \qquad a = (-4,3)$$



- $\nabla f = (2x, 2y)$
- $\nabla f(-4,3) = (-8,6)$
- $D_{\vec{v}} f(-4,3) = (-8,6) \cdot \underbrace{\frac{(-4,3)}{5}}_{\text{scalar}} = \frac{(-8)\cdot(-4) + 6\cdot 3}{5} =$

$$= \frac{32+18}{5} = \frac{50}{5} = 10$$

$f_{\vec{v}}$

$\vec{v_2} = (-8,6)$

$$D_{\vec{v_2}} f(-4,3) = (-8,6) \cdot \frac{(-8,6)}{10} = \frac{64+36}{10} =$$

$$= \frac{100}{10} = 10$$

Solution

$$|(-4,3)| = \sqrt{(-4)^2 + 3^2} = \sqrt{25} = 5$$

$$|(-8,6)| = \sqrt{64+36} = \sqrt{100} = 10$$

Chain rule

$f(x,y) = xy - 2y^3$

$x(u) = e^u$

$y(u) = \ln u \qquad u > 0$

$\frac{df}{du}$

$f(u) = e^u \ln u - 2(\ln u)^3$

$\frac{df}{du} = e^u \ln u + e^u \cdot \frac{1}{u} - 2 \cdot 3 (\ln u)^2 \cdot \frac{1}{u}$

$\frac{df}{du} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial u} + \frac{\partial f}{\partial y} \cdot \frac{dy}{du}$

$\frac{\partial f}{\partial x} = y$

$\frac{\partial f}{\partial y} = x - 6y^2$

$\frac{\partial x}{\partial u} = e^u \qquad \frac{dy}{du} = \frac{1}{u}$

$\frac{df}{du} = y \cdot e^u + (x - 6y^2) \cdot \frac{1}{u}$

$= \ln u \cdot e^u + (e^u - 6\ln^2 u)\frac{1}{u}$

$f(x,y)$

$\frac{\partial f}{\partial x}(2,-4) = 1 \qquad \frac{\partial f}{\partial y}(2,-4) = 5$

$x(t) \qquad x(1) = 2 \qquad x'(1) = 3$

$y(t) \qquad y(1) = -4 \qquad y(1) = -7$

$\frac{df}{dt}(1) = \frac{\partial f}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt}$

$= 1 \cdot 3 + 5 \cdot (-7) = 3 - 35$

$= -32$

$f(x(1), y(1)) = f(2,-4)$