$$f(x,y) = 2y^2 - 4xy + x^4 + 3$$

(1) $\frac{\partial f}{\partial x} = -4y + 4x^3$

$\frac{\partial f}{\partial y} = 4y - 4x$

(2) $-4y + 4x^3 = 0$

$4y - 4x = 0$

$y = x$

$-4x + 4x^3 = 0$

$4x(-1 + x^2) = 0$

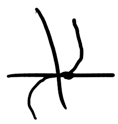
$x_1 = 0$ $\quad x_2 = 1$ $\quad x_3 = -1$

$y_1 = 0$ $\quad y_2 = 1$ $\quad y_3 = -1$

$[0,0]$
$[1,1]$
$[-1,-1]$

(3) $\begin{pmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y^2} \end{pmatrix}$. Hess

$\cup$ $\cap$



saddle point

$$\begin{pmatrix} 12x^2 & -4 \\ -4 & 4 \end{pmatrix}$$

(4) $[\underset{x}{0}, \underset{y}{0}]$

$\begin{pmatrix} 0 & -4 \\ -4 & 4 \end{pmatrix}$ $\quad D_1 = 0$

$D_2 = -16$

Indef → Saddle point

$[1,1]$

$\begin{pmatrix} 12 & -4 \\ -4 & 4 \end{pmatrix}$ $\quad D_1 = 12 > 0$

$D_2 = 48 - 16 = 32 > 0$

Pos def. → loc. min.

$[-1,-1]$ $\begin{pmatrix} 12 & -4 \\ -4 & 4 \end{pmatrix}$ ↑ the same

$$\begin{vmatrix} 0-\lambda & -4 \\ -4 & 4-\lambda \end{vmatrix} = -\lambda(4-\lambda) - 16$$

$$= -16 - 4\lambda + \lambda^2$$

$$\lambda^2 - 4\lambda - 16$$

$$\lambda_{12} = \frac{4 \pm \sqrt{16 - 4\cdot(-16)}}{2}$$

$\sqrt{80} \approx 8{,}9$

$$\lambda_{12} = \frac{4 \pm \sqrt{5\cdot 16}}{2}$$

$$\lambda_1 \approx \frac{13}{2} > 0$$

$$\lambda_2 = \frac{-5}{2} < 0$$

Indef. :)