Implicit function . Domain $\mathbb{R}^2$

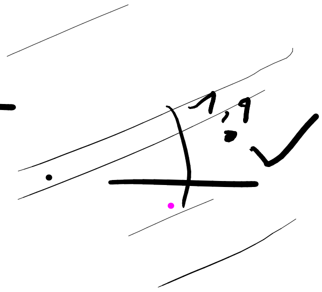
$F(x,y) = x^2+y^2-1-y\sqrt[3]{x^2} \rightarrow$

$F(x,y)=0$ $\qquad x^2+y^2-1-y\sqrt[3]{x^2}=0$

$x^{2/3}$

point $(1,1)$

- $F \in C^1(G)$ ✓ $\qquad G := \mathbb{R}^2$



- $F(1,1) \stackrel{?}{=} 0$

$$1^2+1^2-1-1\cdot\sqrt[3]{1^2}=0 \checkmark$$

- $$\frac{\partial F}{\partial y} = 0+2y-0-1\sqrt[3]{x^2}$$

$$\frac{\partial F}{\partial y}(1,1) = 2\cdot 1 - 1\cdot\sqrt[3]{1} = 1 \neq 0 \checkmark$$

- $$\frac{dy}{dx}(1,1) = -\frac{\frac{\partial F}{\partial x}(1,1)}{\frac{\partial F}{\partial y}(1,1)} = -\frac{\frac{4}{3}}{1} = -\frac{4}{3}$$

$$\frac{\partial F}{\partial x} = 2x - y\,\frac{2}{3}\cdot x^{-1/3}$$

$$\frac{\partial F}{\partial x}(1,1) = 2\cdot 1 - 1\cdot\frac{2}{3}\cdot 1 = \frac{4}{3}$$

$$x^2 + y^2 - 1 - \underline{y} \cdot \underline{x}^{2/3} = 0$$

$$y = y(x)$$

$$2x + \underline{2y \cdot y'} - 0 - \left(y' \cdot x^{2/3} + y \cdot \tfrac{2}{3} x^{-1/3}\right) = 0$$

$$2y(x) \cdot y'(x)$$

$$(\sin x)'$$

$$\left((\underline{\sin x})^2\right)' = 2 \sin x \cdot \cos x$$

$$\left((\underline{y(x)})^2\right)' = 2y(x) \cdot y'(x)$$

$$y'\left(2y - x^{2/3}\right) = -2x + \tfrac{2}{3} y x^{-1/3}$$

$$y' = \frac{-2x + \frac{2}{3} y x^{-1/3}}{2y - x^{2/3}}$$

$x \qquad y = y(x)$

$(1, 1)$

$$y' = \frac{-2 \cdot 1 + \frac{2}{3} \cdot 1 \cdot 1^{-1/3}}{2 \cdot 1 - 1^{2/3}} = \frac{-4/3}{1}$$