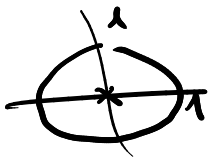
Lagrange multiplier

$f(x,y) = x^2+y$

$0^2+0^2 \neq 1$ ∴

$x^2+y^2 = 1$



$g(x,y) = x^2+y^2-1$

$G = \mathbb{R}^2$

$H = \{ [x,y] \in \mathbb{R}^2 : x^2+y^2-1 = 0 \}$

(1) $\nabla g = 0$

$\nabla g = (2x, 2y) = (0, 0)$

$x = 0$ & $y = 0$ $\quad [0,0]$

∴ $\notin H$

(2) $\frac{\partial f}{\partial x} = 2x \qquad \frac{\partial g}{\partial x} = 2x$

$2x + \lambda 2x = 0$

$\frac{\partial f}{\partial y} = 1 \qquad \frac{\partial g}{\partial y} = 2y$

$1 + \lambda 2y = 0$

—

$2x + \lambda 2x = 0$

$1 + \lambda 2y = 0$

$x^2 + y^2 = 1$

$2x(1+\lambda) = 0$

$x = 0 \qquad \lambda = -1$

$x=0$ $\quad y^2=1 \quad y=\pm 1 \quad [0,1] \quad [0,-1]$

$\lambda=-1 \quad 1-2y=0 \quad y=\frac{1}{2}$

$x^2+y^2=1 \quad x^2+\frac{1}{4}=1$

$x^2=\frac{3}{4} \quad x=\pm\sqrt{\frac{3}{4}}$

$\left[\frac{\sqrt{3}}{4},\frac{1}{2}\right] \quad \left[-\frac{\sqrt{3}}{4},\frac{1}{2}\right]$

Conclusion: $[0,1]$, $[0,-1]$

$\left[\frac{\sqrt{3}}{4},\frac{1}{2}\right]$, $\left[-\frac{\sqrt{3}}{4},\frac{1}{2}\right]$

Lagrange multipliers

$f(x,y,z)=x+2y+3z$

$x-y+z=1$ → plane

$x^2+y^2=1$ ← cylinder

$g_1(x,y,z)=x-y+z-1$

$g_2(x,y,z)=x^2+y^2-1$

$M=\{g_1=0,\ g_2=0\}$

Lin. dep?

(1) $\nabla g_1=(1,-1,1)$ l.i.

$\nabla g_2=(2x,2y,0)$

$1z=2x \quad 1\cdot z=0 \quad z=0$

$-1z=2y \quad \rightarrow x=y=0$ ∴

$f \qquad g_1 \qquad g_2$

$dx: \quad 0 = 1 + 1\lambda_1 + 2x\lambda_2$

$dy: \quad 0 = 2 + -1\lambda_1 + 2y\lambda_2$

$dz: \quad 0 = 3 + 1\lambda_1 + 0\lambda_2$

$x - y + z = 1$

$x^2 + y^2 = 1$

$\lambda_2 \neq 0$ ✗

$0 = 3 + \lambda_1 \rightarrow \lambda_1 = -3$

$0 = -2 + 2x\lambda_2 \qquad 1 = x\lambda_2$

$0 = 5 + 2y\lambda_2 \qquad -\frac{5}{2} = y\lambda_2$

$x - y + z = 1$

$x^2 + y^2 = 1$ ←

$x = \frac{1}{\lambda_2}$

$y = \frac{-5}{2\lambda_2}$

$\frac{1}{\lambda_2^2} + \frac{25}{4\lambda_2^2} = 1$

$4 + 25 = 4\lambda_2^2 \rightarrow \lambda_2^2 = \frac{29}{4}$

$\lambda_2 \overset{\pm}{=} \frac{\sqrt{29}}{2}$

Conclusion:

$\left[\frac{2}{\sqrt{29}}, \frac{-5}{\sqrt{29}}, 1 - \frac{7}{\sqrt{29}}\right]$

$\left[\frac{-2}{\sqrt{29}}, \frac{5}{\sqrt{29}}, 1 + \frac{7}{\sqrt{29}}\right]$ ✓