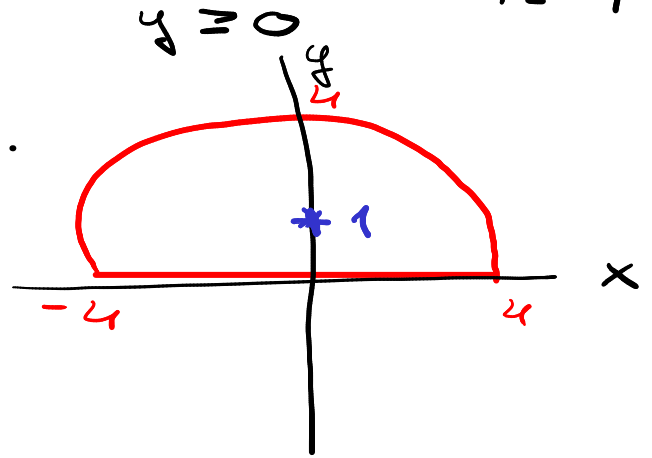
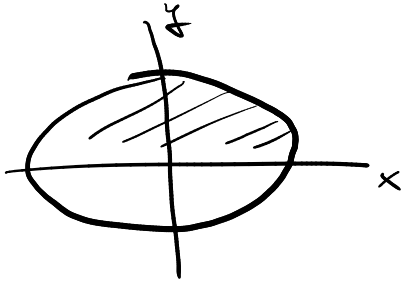


$$f(x,y) = -x^2 - y^2 + 2y$$

$$M: x^2 + y^2 \leq 16$$

$$r = 4$$

(1)



(2) Existenz

$f(x,y)$ is cont. (polynomial)

M compact?

M : bounded \checkmark (radius ≤ 4)
 closed \checkmark (it has boundary) $\} \text{ compact } \checkmark$

(3)(a) int M



$$\frac{\partial f}{\partial x} = -2x = 0 \quad x = 0$$

$$\frac{\partial f}{\partial y} = -2y + 2 = 0 \quad y = 1 \quad \underline{[0, 1]}$$

(b) border

M_1



Lagrange multiplier λ

$$f = -x^2 - y^2 + 2y$$

$$g(x,y) = x^2 + y^2 - 16$$

• $\nabla g = 0$

$(2x, 2y) \rightarrow [0, 0] \notin M_1$
 \vdots

• $\nabla f + \lambda \nabla g = 0$

$$-2x + \lambda 2x = 0$$

$$-2y + 2 + \lambda 2y = 0$$

$$2x(-1 + \lambda) = 0$$

$$\swarrow$$

$$x = 0$$

$$\searrow$$

$$\lambda = 1 \rightarrow$$

$$-2y + 2 + 2y = 0$$

$$2 = 0 \quad \ddagger$$

$$x^2 + y^2 = 16$$

$$y^2 = 16$$

$$y = \pm 4$$

$$[0, 4] \checkmark$$

$$[0, -4] \notin H_1 \quad \ddagger$$

(e) H_2



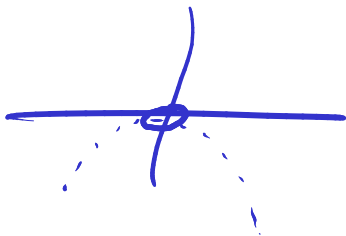
$$f_{H_2} = -x^2 - y^2 + 2y$$

$$y = 0 \quad x \in (-4, 4)$$

$$\underline{f(x, 0) = -x^2 - 0^2 + 2 \cdot 0}$$

$$x = 0 \quad y = 0$$

$$[0, 0] \ddagger$$



glob.

MAX

glob.

MIN

(4) $[0, 0]$

$$[0, 4]$$

$$[0, 1]$$

$$[-4, 0] \quad [4, 0]$$

$$f(x, y)$$

$$\downarrow$$

$$0$$

$$\downarrow$$

$$-16 + 8$$

$$-8$$

$$\downarrow$$

$$-1 + 2$$

$$1$$

$$\downarrow$$

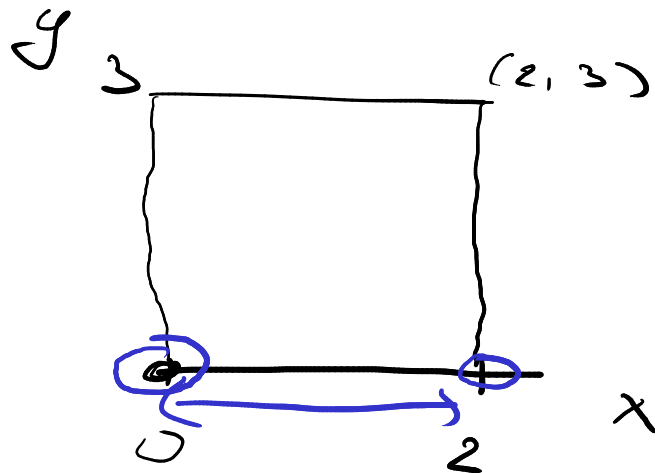
$$-16$$

$$\downarrow$$

$$-16$$

1a $f = x^2 - xy + y$

M: $0 \leq x \leq 2$
 $0 \leq y \leq 3$



- f cont
- M comp.

- int M ✓

- $y = 0 \quad x \in (0, 2)$

$A = [0, 0]$

$B = [2, 0]$

$t \in (0, 1)$

$x = 0 + t(2 - 0) = 2t$

$y = 0 + t(0 - 0) = 0$

$\parallel [0, 0]$

$[2, 0] \checkmark$

$f(2t, 0) = 4t^2 + 0 + 0$

$t = 0$

$[0, 0]$

