Partial fractions

$\int \frac{\text{polynomial}}{\text{polynomial}}$

$\int \frac{1}{x} =$ $\int \frac{1}{x^2} = \int x^{-2} = \frac{x^{-1}}{-1} = \frac{-1}{x}$

(1) $\int \frac{2x^3+11x^2+13x+1}{(x^2+x-2)(x+2)}\,dx$ steps $= \int 2 - \frac{1}{(x+2)^2} + \frac{3}{x-1} + \frac{2}{x+2}\,dx$

$x \neq -2$

$x \neq 1$

$= 2x - \frac{-1}{x+2} + 3\ln|x-1| + 2\ln|x+2| + k$

- $\frac{P}{Q}$ $\log P < \log Q$?

$3 \not< 3$

$x^3+2x^2+x^2+2x-2x-4$

$(2x^3+11x^2+13x+1) : (x^3+3x^2-4) = 2$

$-(2x^3+6x^2-8)$

$5x^2+13x+9$

$\int 2 + \frac{5x^2+13x+9}{(x^2+x-2)(x+2)}\,dx$

- $(x^2+x-2)(x+2) = \underbrace{(x-1)(x+2)}_{x^2+2x-x-2}(x+2)$

$\int 2 + \frac{5x^2+13x+9}{(x-1)(x+2)^2}\,dx$

Remark: $2x+3$, (x^2+1), x^2+x+1

$= \int 2 + \frac{3}{x-1} + \frac{2}{x+2} - \frac{1}{(x+2)^2}$

- Decomposition

$\frac{5x^2+13x+9}{(x-1)(x+2)^2} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$

$A = 3$, $B = 2$, $C = -1$

$$\frac{5x^2+13x+9}{(x-1)(x+2)^2} = \frac{A(x+2)^2 + B(x-1)(x+2) + C(x-1)}{(x-1)(x+2)^2}$$

links

$$5x^2+13x+9 = A(x+2)^2 + B(x-1)(x+2) + C(x-1)$$

C $\quad$ $g(x)$ $\quad$ C

$x=1$: $\quad 5+13+9 = A\cdot 9 + B\cdot 0 + C\cdot 0$

$27 = 9A \rightarrow \boxed{A=3}$

$x=-2$ $\quad 5\cdot 4 + 13(-2) + 9 = A\cdot 0 + B\cdot 0 + C(-3)$

$20 - 26 = -3C$

$3 = -3C \quad \boxed{C=-1}$

$x=0$ $\quad 9 = 3\cdot 4 + B(-2) + (-1)(-1)$

$9 = 13 - 2B$

$-4 = -2B \quad \boxed{B=+2}$

$$\int \frac{5x^2+3x+8}{(x+2)(x^2-2x+3)}\,dx$$

- $\frac{P}{Q}$ st P $\overset{?}{\leq}$ st Q
  $2 < 3$ ✓

- brackets $D = 4 - 4\cdot 1 \cdot 3 < 0$:) ✓

- $$\frac{5x^2+3x+8}{(x+2)(x^2-2x+3)} = \frac{A}{x+2} + \frac{Bx + C}{x^2-2x+3}$$
  (above A: 2; above $Bx + C$: $3x + 1$)
  ↳ quadr.

$$5x^2+3x+8 = A(x^2-2x+3) + (Bx+C)(x+2)$$

- $$5x^2+3x+8 = Ax^2 - 2Ax + 3A + Bx^2 + 2Bx + Cx + 2C$$

x^2: $5 = A + B$

x: $3 = -2A + 2B + C$

1: $8 = 3A + 2C$

→ $B = 5 - A$

$3 = -2A + 10 - 2A + C$
$8 = 3A + 2C$

$-7 = -4A + C$ $|(-2)$
$8 = 3A + 2C$

$14 = 8A - 2C$
$8 = 3A + 2C$

$22 = 11A$ $\boxed{A=2}$

$-7 + 4A = C$

$\boxed{C=1}$ $\boxed{B=3}$

$$\int \frac{2}{x+2} + \frac{3x+1}{x^2-2x+3}\,dx$$

$$\int \frac{2}{x+2} = 2\ln|x+2| + 2$$

$$\int \frac{3x+1}{x^2-2x+3}\,dx = \frac{3}{2}\int \frac{2}{3}\,\frac{3x+1}{x^2-2x+3}\,dx = \frac{2}{3}\int \frac{2x+\frac{2}{3}}{x^2-2x+3}$$

$$= \frac{2}{3}\int \frac{2x-2+2+\frac{2}{3}}{x^2-2x+3}\,dx$$

$$\int \frac{2x-2}{x^2-2x+3}\,dx = \int \frac{1}{y}\,dy = \ln|y| + c = \ln|x^2-2x+3| + c$$

$y = x^2-2x+3$

$dy = 2x-2\,dx$

$$\frac{2}{3}\int \frac{2x-2}{x^2-2x+3}\,dx = \frac{2}{3}\ln|x^2-2x+3| + c$$

$$\frac{2}{3}\int \frac{2/3}{x^2-2x+3} = \frac{16}{9}\int \frac{1}{x^2-2x+3}\,dx$$

$x^2-2x+3 = (x-1)^2 + 2$

x^2-2x

$(x-a)^2$

$x^2-2ax+a^2$

$\Rightarrow a=1$

$(x-1)^2$

x^2-2x+1

$$= \frac{16}{9}\int \frac{1}{(x-1)^2+2}\,dx = \frac{16}{9}\int \frac{1}{2\left[\frac{(x-1)^2}{2}+1\right]}\,dx$$

need 1

$$= \frac{8}{9} \int \frac{1}{\left(\frac{x-1}{\sqrt{2}}\right)^2 + 1} \, dx = \frac{8}{9} \sqrt{2} \arctan \frac{x-1}{\sqrt{2}} + C$$

$$\int \frac{1}{1+x^2} = \arctan x$$

Together

$$\int = 2 \ln|x+2| - \frac{2}{3} \ln|x^2 - 2x + 3| + \frac{8\sqrt{2}}{9} \arctan \frac{x-1}{\sqrt{2}} + C$$

$x \neq -2$