5th lesson - Dividing polynomials

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Theory

Theorem 1. Let $f(x) = a_n x^n + \cdots + a_1 x + a_0$ be a function with $a_i \in \mathbb{Z}$, $i = 0, \ldots, n$. Then every rational root of f is of the following form

$$\frac{p}{q} = \pm \frac{\text{factor of } a_0}{\text{factor of } a_n}.$$

Exercises

1. Divide

   (a) $\dfrac{x^2 - 4x - 12}{x - 6}$

   (b) $\dfrac{2x^3 - 3x^2 - 19x + 30}{x - 3}$

   (c) $\dfrac{2x^3 + 8x^2 - 3x - 12}{x + 4}$

   (d) $\dfrac{2x^3 - 8x^2 + 9x - 2}{x - 2}$

   (e) $\dfrac{8x^3 + 14x + 8}{2x + 1}$

   (f) $\dfrac{27x^3 + 9x^2 - 3x - 10}{3x - 2}$

   (g) $\dfrac{4x^3 - 8x^2 - 3x + 1}{x + \frac{1}{2}}$

   (h) $\dfrac{x^4 + x^3 + 7x^2 - 6x + 8}{x^2 + 2x + 8}$

2. Divide

   (a) $\dfrac{5x^3 - x^2 + 6}{x - 4}$

   (b) $\dfrac{3x^4 - 5x^2 + 3}{x + 2}$

   (c) $\dfrac{x^3 + 2x^2 - 3x + 4}{x - 7}$

   (d) $\dfrac{2x^5 + x^4 - 6x + 9}{x^2 - 3x + 1}$

   (e) $\dfrac{4x^3 - 3x - 2}{x + 1}$

   (f) $\dfrac{6x^2 - x - 2}{x + 1}$

   (g) $\dfrac{3x^4 + 2x^3 + x^2 + 4}{x^2 + 1}$

   (h) $\dfrac{27x^3 + 9x^2 - 3x - 9}{3x - 2}$

   Find all roots and factor polynomials

3. (a) $x^3 + 2x^2 - 11x - 12$

   (b) $10x^4 - 3x^3 - 29x^2 + 5x + 12$

   (c) $2x^3 - 13x^2 + 3x + 18$

   (d) $x^4 - 3x^3 - 5x^2 + 3x + 4$

   (e) $2x^4 - 7x^3 - 2x^2 + 28x - 24$

   (f) $8x^5 + 36x^4 + 46x^3 + 7x^2 - 12x - 4$

4. You are designing a candle-making kit. Each kit will contain 25 cubic inches of candle wax and a mold for making a model of the pyramid-shaped building at the Louvre Museum in Paris, France. You want the height of the candle to be 2 inches less than the length of each side of the candle's square base. What should the dimensions of your candle mold be?

   Source:https://www.classzone.com/eservices/home/pdf/student/LA206FAD.pdf