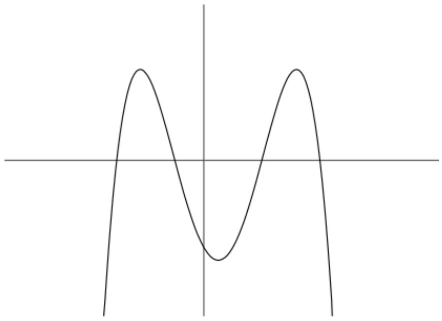
6th lesson

https://www2.karlin.mff.cuni.cz/~kuncova/en/teachIM.php, kunck6am@natur.cuni.cz

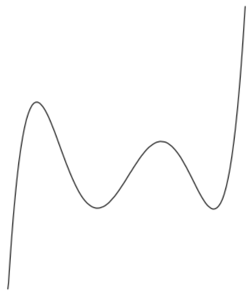
Main source: http://mathquest.carroll.edu/precalc.html

1. The sketched polynomial is of:
   (a) odd degree, lead coefficient negative
   (b) odd degree, lead coefficient positive
   (c) even degree, lead coefficient negative
   (d) even degree, lead coefficient positive



   Solution: (c)

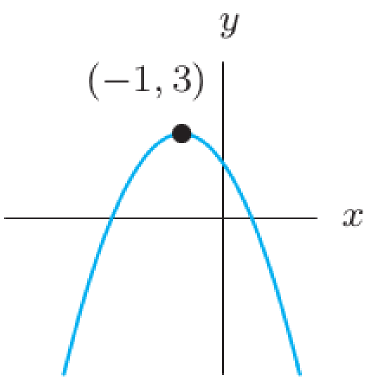
2. The sketched polynomial is of:
   (a) odd degree, lead coefficient negative
   (b) odd degree, lead coefficient positive
   (c) even degree, lead coefficient negative
   (d) even degree, lead coefficient positive



   Solution: (b)

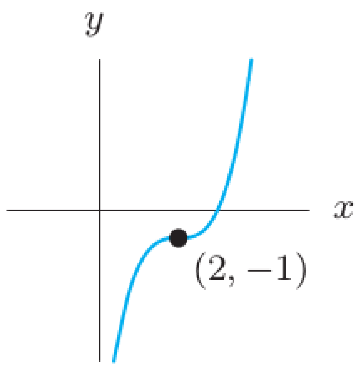
   Source for 4,5: Calculus: Single and Multivariable, 6th Edition, Deborah Hughes-Hallett, Andrew M. Gleason, William G. McCallum

3. The sketched polynomial is:
   (a) $(x-1)^2+3$
   (b) $-(x+3)^2-1$
   (c) $(x-3)^2+1$
   (d) $(x+3)^2-1$
   (e) $-(x+1)^2+3$



   Solution: (e)

4. The sketched polynomial is:



   (a) $(x-2)^3-1$
   (b) $(x+2)^3-1$
   (c) $(x+2)^3+1$
   (d) $(x-2)^3+1$
   (e) $-(2-x)^3-1$

   Solution: (a), (e)

5. What is the degree of the polynomial $y = x(2x+1)^3(x-4)^2(5-x)^5$?

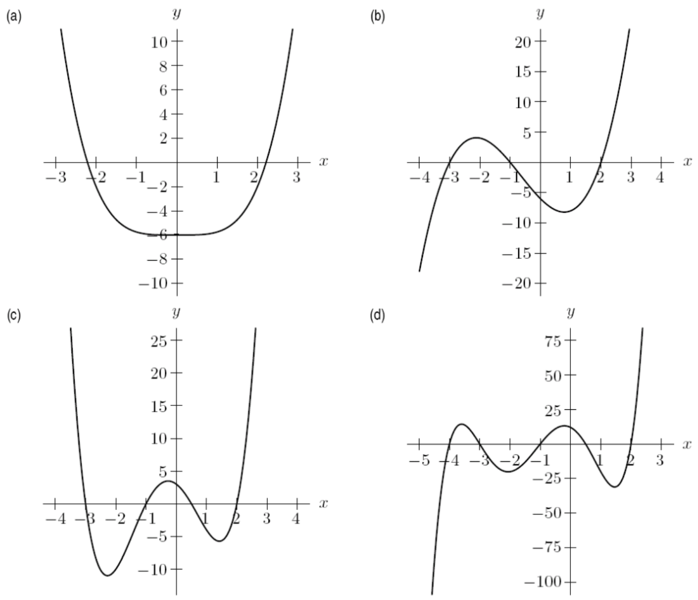
   Solution: 11

6. Find the polynomial with the smallest possible degree, with zeros at $x = 1$, $x = 2$ and $x = 3$ such that $f(5) = 8$.

   (a) $(x-1)(x-2)(x-3)$
   (b) $(x-1)(x-2)(x-3)(x-5)$
   (c) $8(x-1)(x-2)(x-3)$
   (d) $8(x-1)(x-2)(x-3)(x-5)$
   (e) $\frac{1}{3}(x-1)(x-2)(x-3)$
   (f) $\frac{1}{42}(x-1)(x-2)(x-3)$

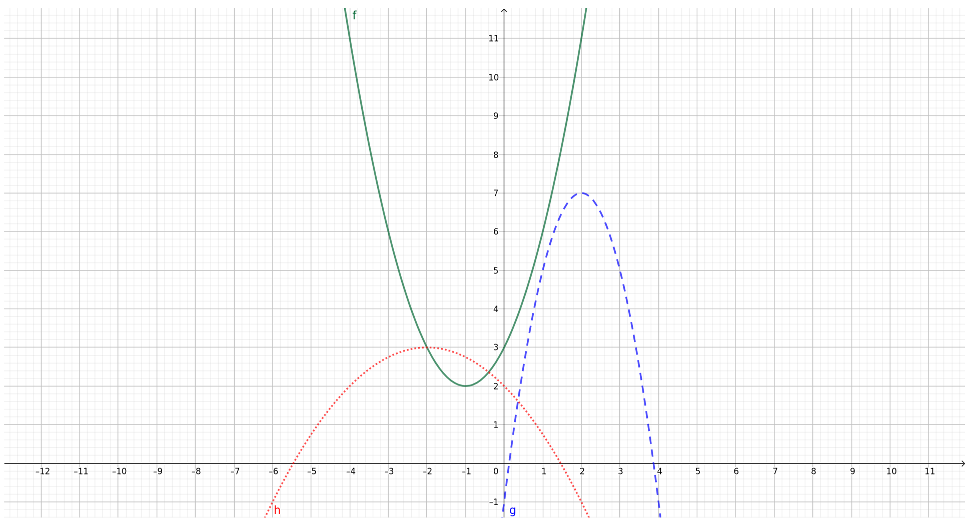
   Solution: (e)

7. Find the graph of the function $y = x^3 + 2x^2 - 5x - 6$



Solution: (b)

8. Find the formula for the quadratic functions:



Solution: green: $(x+1)^2+2$, red dotted: $\frac{-(x+2)^2+12}{4}$, blue dashed: $-2(x-2)^2+7$

9. Decide

TRUE If $f(x)$ is a polynomial such that $f(c) = 0$ for $c \in \mathbb{R}$, then $f(x)$ can be written as $(x - c)g(x)$ for some polynomial $g(x)$.

FALSE A polynomial function may have a horizontal asymptote.

FALSE A polynomial function may have a vertical asymptote.

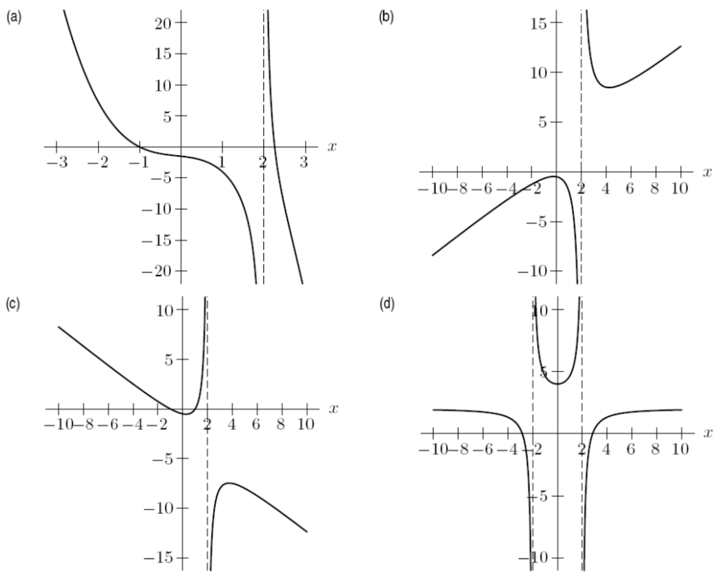
FALSE For $x \in \mathbb{R}$ we have: $x \leq x^2$.

FALSE Every polynomial of even degree is an odd function and every polynomial of odd degree is even function.

FALSE Every polynomial of even degree is an even function and every polynomial of odd degree is odd function.

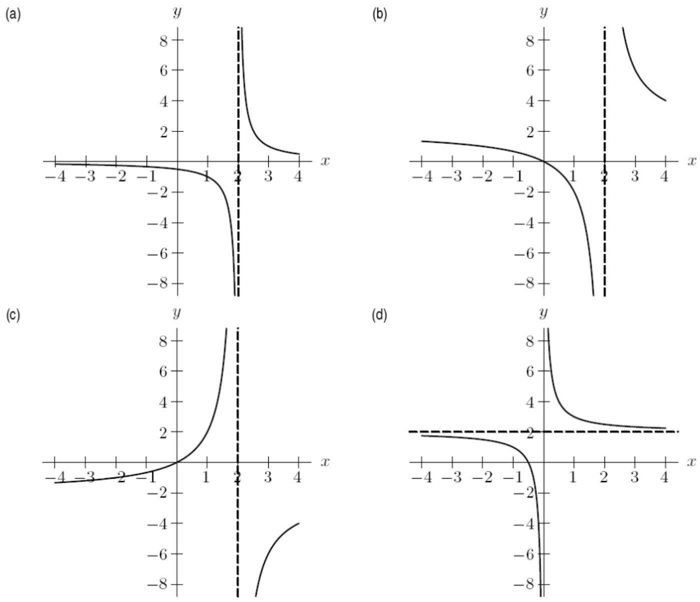
FALSE Let $f(x) = \frac{x^2-1}{x+1}$, $g(x) = x - 1$. Then $f(x) = g(x)$.

10. Find the graph of the function $y = \frac{1-x^2}{x-2}$



Solution: (c)

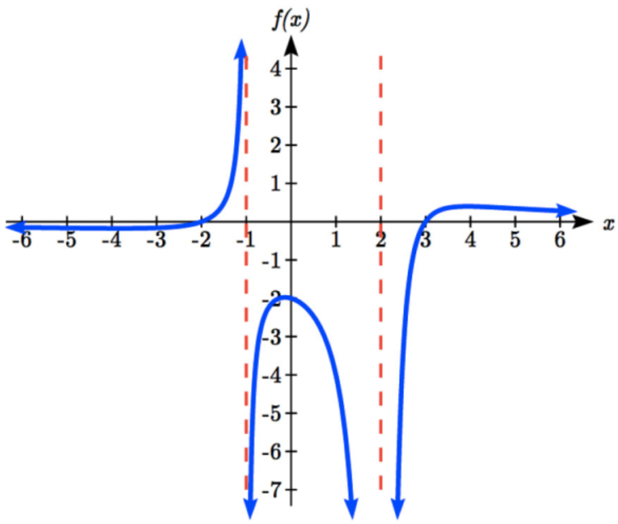
11. Find the graph of the function $y = \frac{2x}{x-2}$



Solution: (c)

Source:`http://www.opentextbookstore.com/precalc/2/Precalc3-7.pdf`

12. Find the possible formulas for graphed functions.



$$\frac{4(x+2)(x-3)}{3(x+1)(x-2)^2}$$

Try it Now

5. Given the function $f(x)=\dfrac{(x+2)^2(x-2)}{2(x-1)^2(x-3)}$, use the characteristics of polynomials and rational functions to describe its behavior and sketch the function.

Since a rational function written in factored form will have a horizontal intercept where each factor of the numerator is equal to zero, we can form a numerator that will pass through a set of horizontal intercepts by introducing a corresponding set of factors. Likewise, since the function will have a vertical asymptote where each factor of the denominator is equal to zero, we can form a denominator that will produce the vertical asymptotes by introducing a corresponding set of factors.

Writing Rational Functions from Intercepts and Asymptotes

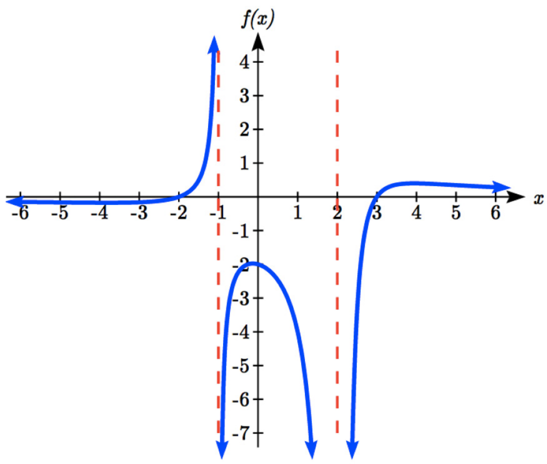
If a rational function has horizontal intercepts at $x = x_1, x_2, \ldots, x_n$, and vertical asymptotes at $x = v_1, v_2, \ldots, v_m$ then the function can be written in the form

$$f(x)=a\frac{(x-x_1)^{p_1}(x-x_2)^{p_2}\cdots(x-x_n)^{p_n}}{(x-v_1)^{q_1}(x-v_2)^{q_2}\cdots(x-v_m)^{q_n}}$$

where the powers p_i or q_i on each factor can be determined by the behavior of the graph at the corresponding intercept or asymptote, and the stretch factor a can be determined given a value of the function other than the horizontal intercept, or by the horizontal asymptote if it is nonzero.

Example 10

Write an equation for the rational function graphed here.



The graph appears to have horizontal intercepts at $x = -2$ and $x = 3$. At both, the graph passes through the intercept, suggesting linear factors.

The graph has two vertical asymptotes. The one at $x = -1$ seems to exhibit the basic behavior similar to $\dfrac{1}{x}$, with the graph heading toward positive infinity on one side and heading toward negative infinity on the other.

The asymptote at $x = 2$ is exhibiting a behavior similar to $\frac{1}{x^2}$, with the graph heading toward negative infinity on both sides of the asymptote.

Utilizing this information indicates an function of the form
$$f(x) = a\frac{(x+2)(x-3)}{(x+1)(x-2)^2}$$

To find the stretch factor, we can use another clear point on the graph, such as the vertical intercept (0,-2):
$$-2 = a\frac{(0+2)(0-3)}{(0+1)(0-2)^2}$$
$$-2 = a\frac{-6}{4}$$
$$a = \frac{-8}{-6} = \frac{4}{3}$$

This gives us a final function of $f(x) = \frac{4(x+2)(x-3)}{3(x+1)(x-2)^2}$

Oblique Asymptotes

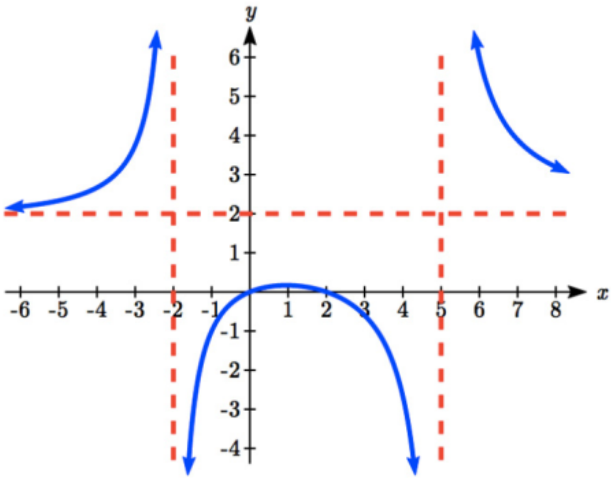
Earlier we saw graphs of rational functions that had no horizontal asymptote, which occurs when the degree of the numerator is larger than the degree of the denominator. We can, however, describe in more detail the long-run behavior of a rational function.

Example 11

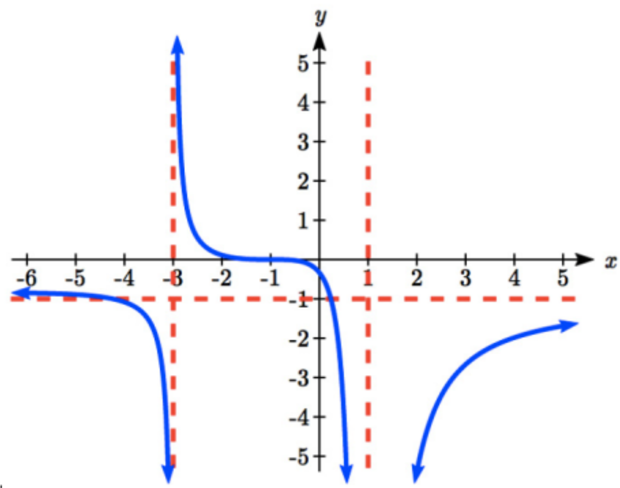
Describe the long-run behavior of $f(x) = \frac{3x^2+2}{x-5}$

Earlier we explored this function when discussing horizontal asymptotes. We found the long-run behavior is $f(x) \approx \frac{3x^2}{x} = 3x$, meaning that $x \to \pm\infty$, $f(x) \to \pm\infty$, respectively, and there is no horizontal asymptote.

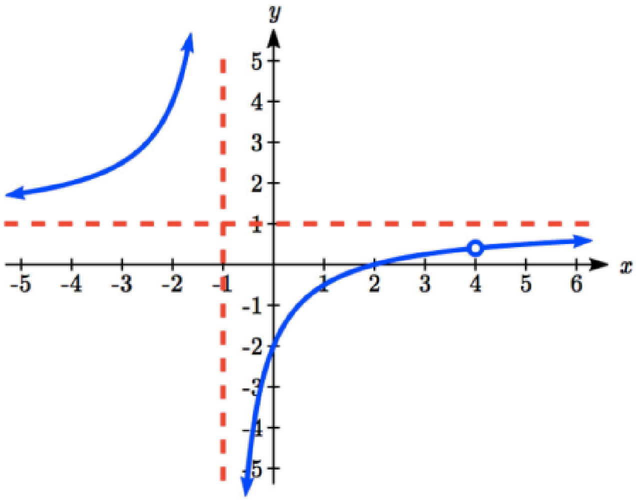
If we were to do polynomial long division, we could get a better understanding of the behavior as $x \to \pm\infty$.



$$\frac{2x(x-2)}{(x+2)(x-5)}$$



$$\frac{-(x+1)^3}{(x+3)(x-1)^2}$$



$$\frac{(x-2)(x-4)}{(x+1)(x-4)}$$