

8th lesson - Goniometric equations

<https://www2.karlin.mff.cuni.cz/~kuncova/en/teaching.php>, kuncova@karlin.mff.cuni.cz

Exercises

1. Solve for $x \in \mathbb{R}$.

Algorithm for $\sin u$, $\cos u$ (and $\tan u$, $\cot u$):

- Are there conditions?
- Find all the solutions u on the interval $[0, 2\pi)$. Use the unit circle or value table and make sure, You have all the solutions.
- Add the constant $+k2\pi$ (or $+k\pi$ for \tan or \cot).
- Express u .

(a) $\sin x = \frac{1}{2}$	(g) $\cos(2x) = -\frac{\sqrt{3}}{2}$
(b) $\cos x = \frac{1}{2}$	(h) $\frac{1}{\sin(\frac{x}{3}-\pi)} = \sqrt{2}$
(c) $\tan x = \sqrt{3}$	(i) $\cot(3x) = 0$
(d) $3\sin x - 2 = 5\sin x - 1$	(j) $\frac{1}{\cos^2 x} = 4$
(e) $2\sin x - 1 = 0$	(k) $\tan \frac{x}{2} = -3$
(f) $2\cos x = \sqrt{2}$	(l) $2\cos(4x) + 1 = 0$

2. Solve for $x \in [0, 2\pi)$

Algorithm:

- Check the conditions.
- Our aim is to have 0 on the right side and a product of functions (or quadratic equation) on the left side.
- Move functions to the left side, factor out everything You can.
- Do not divide by functions (if possible).
- Apply formulae.
- Solve for the product or substitute for the quadratic equation.
- Make a conclusion.

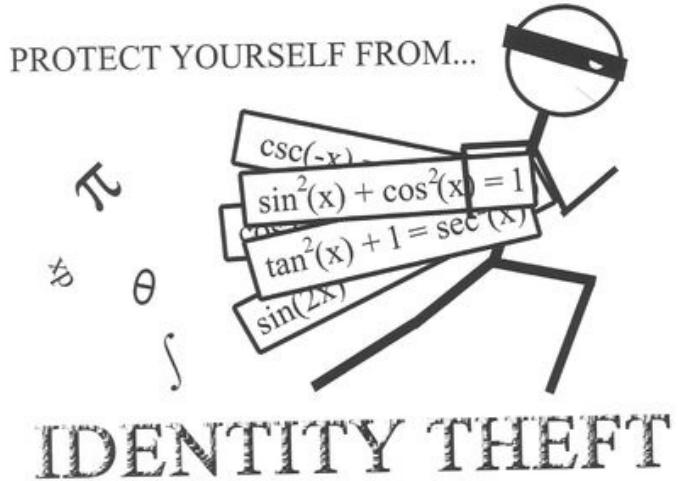
(a) $\sin^2 x \cos x = 4 \cos x$	(h) $2\sin^2 x - 1 = 0$
(b) $3\sin^3 x = \sin^2 x$	(i) $\tan(x - \frac{\pi}{2}) = 1$
(c) $\cos(2x) = 3\cos x - 2$	(j) $2\cos^2 x + \cos x - 1 = 0$
(d) $\sin(2x) = \sqrt{3}\cos x$	(k) $\tan x \sin^2 x = 3\tan x$
(e) $\cos^2 x - 2\cos x = 3$	(l) $2\cos^2 x + 3\sin x = 0$
(f) $\tan^2 x - 2 = 3\tan x$	
(g) $2\sin^2 x + \cos x = 1$	(m) $\cos 2x + 3\sin x - 2 = 0$

3. Solve for $x \in [0, 2\pi)$

Algorithm:

- What about the conditions?
- Modify the inequation as in the previous exercise.
- Do not divide by a function. If You do, consider different cases (it can change the inequation mark)
- Solve for an equation.
- Make a table value / solve with graph / solve with unit circle.
- Make a conclusion.

- | | |
|-------------------------------|--|
| (a) $2 \sin x \leq 1$ | (f) $\sin(2x) > \cos x$ |
| (b) $\tan x \geq 3$ | (g) $-6 \sin x \leq -3$ |
| (c) $\tan x \geq -\sqrt{3}$ | (h) $3 \cot^2 x + 3 \cot x - \sqrt{3} \cot x < \sqrt{2}$ |
| (d) $\sin x \geq \frac{1}{2}$ | (i) $(2 \cos x - 1)(2 \cos x + 1) < 0$ |
| (e) $\sin x > \cos x$ | |



created by: David Ritzenthaler

3.14.2013

1: http://www.geek-jokes.com/identity-theft/?post_in_lightbox=1