12th lesson - Composition of functions

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Exercises

Compositions

1. Let $f(x) = x^2$ and $g(x) = x - 2$. Find

   (a) $f(g(3))$ (b) $g(f(3))$ (c) $f(g(x))$ (d) $g(f(x))$

2. Let $f(x) = 4 - x^2$ and $g(x) = \sqrt{x}$. Find

   (a) $f(g(x))$ (b) $g(f(x))$

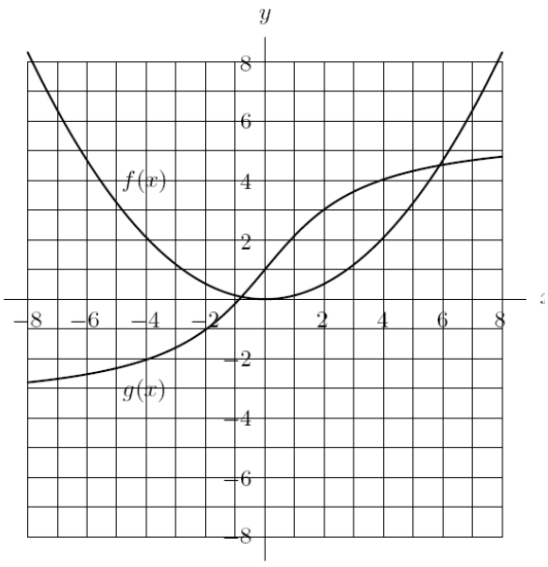
3. Let $f(x) = 3x - 8$ and $g(x) = \frac{x+8}{3}$. Find

   (a) $f(g(x))$ (b) $g(f(x))$

4. Express the following functions as composition:

   (a) $(1 + x^3)^{27}$ (b) e^{-x^2} (c) $-(e^x)^2$

5. Find $g(f(3))$, if the f and g are at the picture:



1: http://nebula2.deanza.edu/~karl/Classes/Files/Math12/ch01.pdf

6. The values of functions f and g can be found in the table. Find $f(g(0))$.

x	-2	-1	0	1	2
$f(x)$	1	0	-2	2	-1
$g(x)$	-1	1	2	0	-2

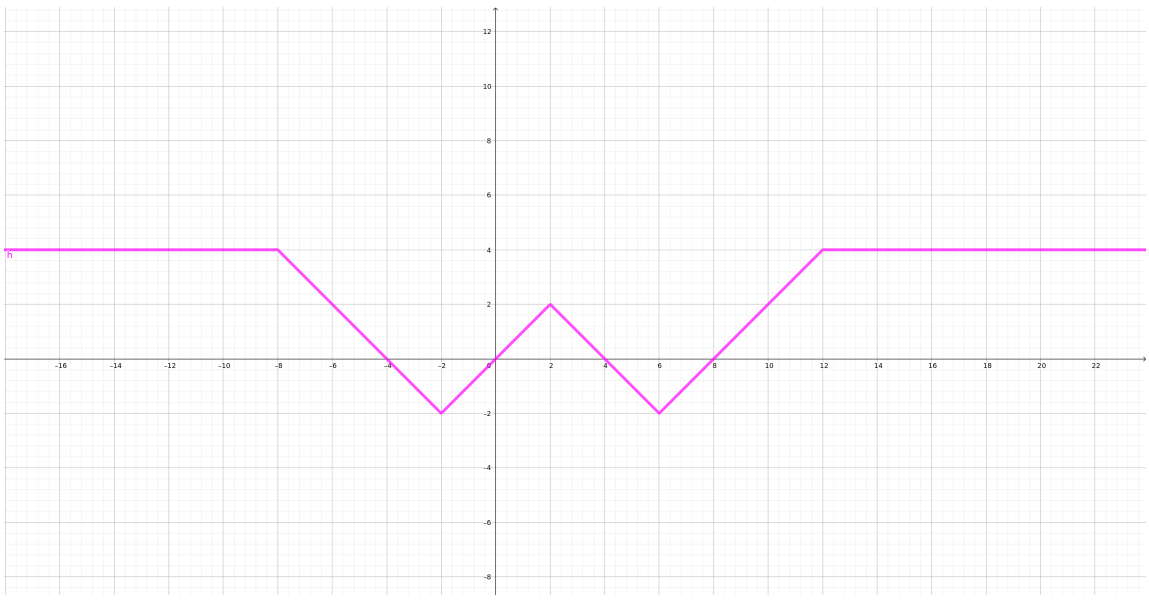
7. The values of functions f and g can be found in the table. Find x, if $f(g(x)) = 1$.

x	-2	-1	0	1	2
$f(x)$	1	0	-2	2	-1
$g(x)$	-1	1	2	0	-2

8. Look at function $h(x)$ (the pink one)
https://www.geogebra.org/calculator/zu6td6rv

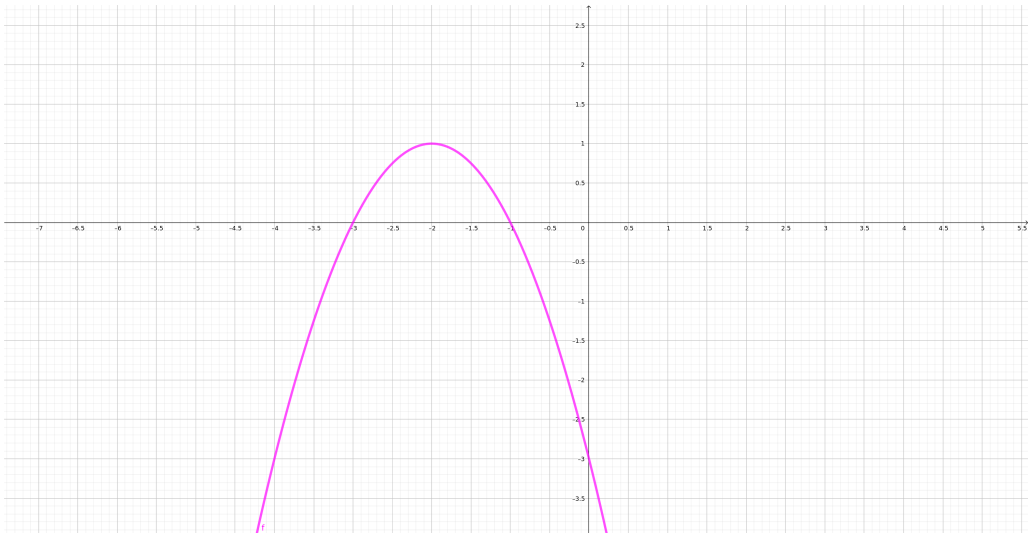
Sketch

(a) $\frac{h(x)}{2}$ (b) $3h(x)$ (c) $-2h(x)$ (d) $h(-x)$ (e) $h(3x)$ (f) $h(x/2)$



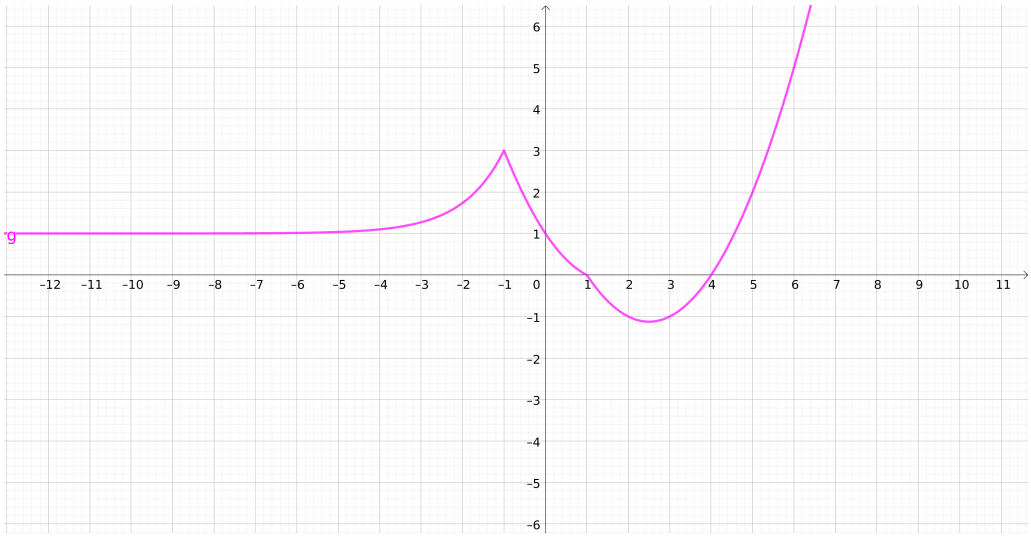
9. Look at function $f(x)$ (the pink one)
https://www.geogebra.org/calculator/rcd6wsup
Sketch

(a) $f(x+1)$ (b) $f(x-1)$ (c) $f(x)+1$ (d) $f(x)-1$

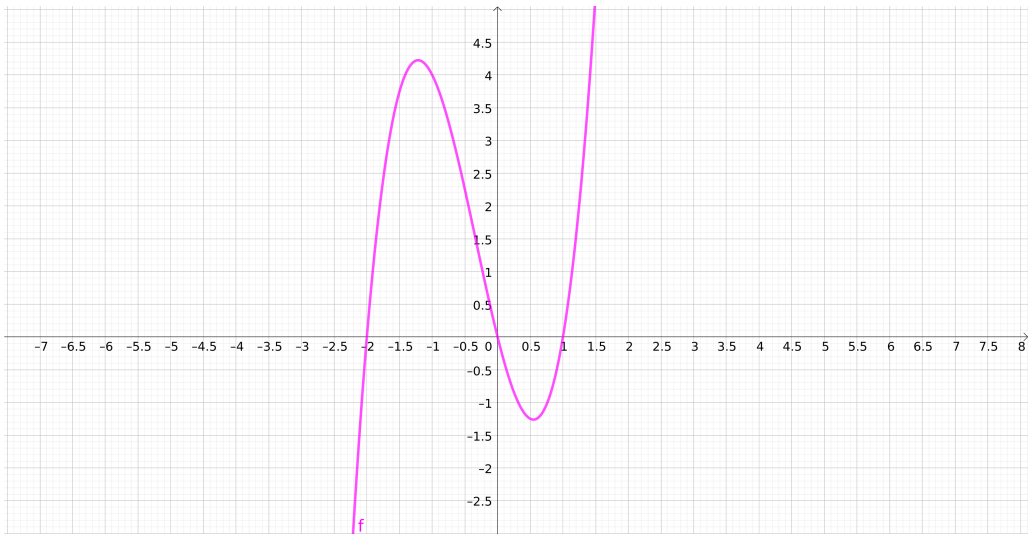


10. Look at function $g(x)$ (the pink one)
https://www.geogebra.org/calculator/sa3h5jad
Sketch

(a) $|g(x)|$ (b) $g(|x|)$ (c) $-g(|x|)$ (d) $g(-|x|)$



11. Look at function $f(x)$ (the pink one)
https://www.geogebra.org/calculator/ksyvvk9z
Sketch $\frac{1}{f(x)}$



Properties

12. Let $f(x)$ and $g(x)$ be functions (with suitable domains and images).
TRUE or FALSE?

(a) Let f and g be odd. Then
   i. $f + g$ is odd.
   ii. fg is odd.
   iii. $f(g)$ is odd.

(b) Let f be even, g be odd. Then
   i. fg is even.
   ii. $f(g)$ is even.
   iii. $g(f)$ is even.
   iv. $g + f$ is even.

(c) Let f and g be increasing. Then
   i. $f + g$ is increasing.
   ii. fg is increasing.
   iii. $f(g)$ is increasing.

(d) Let f be even.
   i. If f is increasing on $(0, \infty)$, then f is increasing also on $(-\infty, 0)$.
   ii. If f is convex on $(0, \infty)$, then f is convex also on $(-\infty, 0)$.