$$\boxed{\lim_{x,y\to\infty} (x^2+y^2)\, e^{-(x+y)}}$$

$$\frac{(x^2+y^2)}{e^{x+y}} = \frac{x^2}{e^{x+y}} + \frac{y^2}{e^{x+y}} < \frac{x^2}{e^x} + \frac{y^2}{e^y} \longrightarrow \boxed{0}$$

$$\boxed{\lim_{x,y\to\infty} \left(\frac{xy}{x^2+y^2}\right)^{x^2}}$$

$$x^2+y^2 \geq 2xy \qquad (x-y)^2 \geq 0$$

$$\Rightarrow \quad \frac{xy}{x^2+y^2} \leq \frac{1}{2}$$

$$0 < \left(\qquad \right)^{x^2} \leq \left(\frac{1}{2}\right)^{x^2} \to \boxed{0}$$

$$\boxed{\lim_{\substack{x\to0\\y\to0}} (x^2+y^2)^{x^2y^2}} \qquad \left[0^0\right]_*$$

$$x^2y^2 \leq \frac{1}{4}(x^2+y^2)^2 \quad , \quad 1 \geq (x^2+y^2)^{x^2y^2} \geq (x^2+y^2)^{\frac{1}{4}(x^2+y^2)^2}$$

$$0 < x^2+y^2 < 1$$

$$\lim_{x,y\to0} (x^2+y^2)^{x^2y^2} = \lim_{t\to0_+} t^{\frac{1}{4}t^2} = \lim_{t\to 0_+} e^{\frac{t^2}{4}\ln t} = \boxed{1}$$

zprava

$$\boxed{\lim_{\substack{y \to \infty \\ x \to \infty}} \frac{x+y}{x^2 - xy + y^2}}$$

$$0 \leq \left| \frac{x+y}{x^2 - xy + y^2} \right| \leq \left| \frac{x+y}{xy} \right| \leq \frac{1}{|y|} + \frac{1}{|x|}$$

$$0 \leq \lim \left| \quad \right| \leq \lim_{\substack{x \to \infty \\ y \to \infty}} \frac{1}{|x|} + \frac{1}{|y|} = \boxed{0}$$

VO AL

(jsme v 0 limitě)

$$\boxed{\lim_{y,x \to \infty} \frac{x^2 + y^2}{x^4 + y^4}}$$

$$0 < \frac{x^2 + y^2}{x^4 + y^4} = \frac{x^2}{x^4 + y^4} + \frac{y^2}{x^4 + y^4} \leq \frac{x^2}{x^4} + \frac{y^2}{y^4} = \frac{1}{x^2} + \frac{1}{y^2} = \boxed{0}$$

$$\boxed{\lim_{\substack{x \to 0 \\ y \to a}} \frac{\sin xy}{x} \qquad a \in \mathbb{R}}$$

$$\frac{\sin xy}{x} = \frac{\sin xy}{xy} \cdot y$$

$$\lim_{\substack{x \to 0 \\ y \to a}} \frac{\sin xy}{xy} = \lim_{t \to 0} \frac{\sin t}{t} = 1$$

$$xy = t, \; a \neq \infty$$

$$\text{pak} \quad \lim_{\substack{x \to 0 \\ y \to a}} \frac{\sin xy}{y} = \lim_{t \to 0} \frac{\sin t}{t} \cdot \lim_{y \to a} y = \boxed{a}$$

$$\boxed{\lim_{\substack{x \to \infty \\ y \to a}} \left(1+\frac{1}{x}\right)^{\frac{x^2}{x+y}}}$$

$$\lim \left(1+\frac{1}{x}\right)^{\frac{x^2}{x+y}} = \lim e^{\frac{1}{1+\frac{y}{x}} \ln\left(1+\frac{1}{x}\right)^x} = \boxed{e}$$

$$\text{neb} \quad \lim \underbrace{\frac{1}{1+\frac{y}{x}}}_{\to 1} \underbrace{\ln \underbrace{\left(1+\frac{1}{x}\right)^x}_{\text{VOAL } \to e}}_{\to 1} \to 1$$

$$\boxed{\begin{array}{c} g: \\ u = xyz \text{ (na) } x^2+y^2+z^2 = 3 \end{array}}$$

hodina

φ f, f spojitá

$$\Phi = xyz + \lambda(x^2+y^2+z^2-3)$$

$$\Phi'_x = yz + 2\lambda x = 0$$

$$\Phi'_y = xz + 2\lambda y = 0$$

$$\Phi'_z = xy + 2\lambda z = 0$$

$$\Phi'_\lambda = x^2+y^2+z^2-3 = 0$$

podm.

$\frac{\partial g}{\partial x}$ $2x = 0$

$\frac{\partial g}{\partial y}$ $2y = 0$

$\frac{\partial g}{\partial z}$ $2z = 0$

$\to (0,0,0) \overset{?}{\neq}$

$(2x, 2y, 2z) = (0,0,0)$

$$xyz + 2\lambda x^2 = 0$$
$$xyz + 2\lambda y^2 = 0$$
$$xyz + 2\lambda z^2 = 0$$

$$3xyz + 2\lambda(x^2+y^2+z^2) = 0$$
$$3xyz = -6\lambda$$
$$xyz = -2\lambda$$

$$\to \quad 2\lambda(x^2-1) = 0$$
$$2\lambda(y^2-1) = 0$$
$$2\lambda(z^2-1) = 0$$

$\to$ • $\lambda = 0$

• $x, y, z = \pm 1 \Rightarrow$

$\lambda = -\frac{1}{2}$

$1 \ 1 \ 1$

$1 \ -1 \ -1$

$-1 \ 1 \ -1$

$-1 \ -1 \ 1$

$\lambda = \frac{1}{2}$

$-1 \ -1 \ -1$

$-1 \ 1 \ 1$

$1 \ -1 \ 1$

$1 \ 1 \ -1$