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(1) $\boxed{f = 1}$ $\quad \mathcal{L}f(x) = \int_0^\infty e^{-xt}\,dt = \left[-\frac{1}{x} e^{-xt}\right]_0^\infty = \lim - \left(-\frac{1}{x}\cdot 1\right)$

$+ \lim_{t\to\infty} -\frac{1}{x} e^{-xt} = \boxed{\frac{1}{x}} \qquad x > 0$

pro $x = 0$ neexistuje (není f)

(2) $\boxed{f = e^{at},\ a \in \mathbb{R}}$ $\quad \int_0^\infty e^{at} e^{-xt}\,dt = \int_0^\infty e^{t(a-x)}\,dt =$

$= \left[\frac{e^{t(a-x)}}{a-x}\right]_0^\infty = \frac{1}{x-a} + \lim_{t\to\infty} e^{t(a-x)}$

$$= \boxed{\begin{cases} \frac{1}{x-a} & x > a \\ \text{nedef} & x \le a \end{cases}}$$

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$f = \sin bt$

(4) $\sin at = \dfrac{e^{iat} - e^{-iat}}{2i}$

položme $a = \pm ib$

pak z předchozího příkladu

$L(e^{ibt}) = \dfrac{1}{x - ib}$ toto platí pro

$L(e^{-ibt}) = \dfrac{1}{x + ib}$ $\operatorname{Re} x > \operatorname{Re}(\pm ib) = |\operatorname{Im} b|$

celkem (linearita)

$$L(\sin at) = \frac{1}{2i}\left(-\frac{1}{x+ib} + \frac{1}{x-ib}\right) = \frac{x-ib-x-ib}{-2i(x^2+b^2)}$$

$$= \underline{\underline{\frac{b}{x^2+b^2}}}$$

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(4)

$f = t^4 e^{at}$

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Derivace dle parametru

víme $L(e^{at}) = \frac{1}{x-a}$

derivujeme podle a → podle parametru

$L(f'(a))(x) = \lambda\, L(f(a))(x)$

$L(t e^{at}) = \frac{1}{(x-a)^2}$

dále

$L(t^2 e^{at}) = \frac{2}{(x-a)^3}$

$L(t^3 e^{at}) = \frac{3!}{(x-a)^4}$

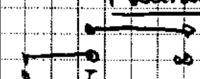
$L(t^4 e^{at}) = \frac{4!}{(x-a)^4}$

(5)

$f = u_\tau$

3

Posunutí



$L(u_0) = \frac{1}{x}$

z věty o posunutí máme $L(u_\tau(t) \cdot 1)(x)$

$= e^{-\tau x} L(1)(x)$

$= e^{-\tau x} \cdot \frac{1}{x}$ $\quad x > 0$

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10) 11 $f(t) = \frac{\sin t}{t}$

víme $L(\sin t) = \frac{1}{x^2+1}$

$$L\left(\frac{\sin t}{t}\right) = \int_x^\infty L(\sin(y))\,dy =$$

$$= \int_x^\infty \frac{1}{1+y^2}\,dy = [\operatorname{arctan} y]_x^\infty =$$

$$= \frac{\pi}{2} - \operatorname{arctan}(x) = \underline{\operatorname{arccotg}(x)}$$

<u>Další příklady</u> ltru

(10) 5

$$f(t) = (2t+5)\cdot e^{-2t} + 3\cos t - 2\sin 3t$$

$$= 2te^{-2t} + 3\cos t + 5e^{-2t} - 2\sin 3t$$

$$2\frac{1}{(x-(-2))^2} + 3\frac{x}{x^2+1} - 2\frac{3}{x^2+9} + 5\frac{1}{x-(-2)}$$

(11) 6

$$f(t) = \begin{cases} 0 & 0 \le t \le 1 \\ e^{-t} & 1 < t \le 2 \\ 0 & t > 2 \end{cases}$$

$$f(t) = e^{-t}(u_1(t) - u_2(t))$$

$$= e^{-(t-1)-1} u_1(t) - e^{-(t-2)-2} u_2(t)$$

$$\Rightarrow e^{-1} e^{-1x} \frac{1}{x+1} - e^{-2} e^{-2x} \frac{1}{x+1}$$

(12) 7

$$f(t) = \begin{cases} \cos t & 0 \le t \le \frac{\pi}{2} \\ 1 & t > \frac{\pi}{2} \end{cases}$$

$$f(t) = \cos t \left(u_0(t) - u_{\frac{\pi}{2}}(t) \right) + u_{\frac{\pi}{2}}(t)$$

$$= \cos t \, u_0(t) - \cos\left(t - \tfrac{\pi}{2} + \tfrac{\pi}{2}\right) u_{\frac{\pi}{2}}(t) + u_{\frac{\pi}{2}}(t)$$

$$u_{\frac{\pi}{2}}(t) \left(\cos(t - \tfrac{\pi}{2}) \cos \tfrac{\pi}{2} - \sin \tfrac{\pi}{2} \sin(t - \tfrac{\pi}{2}) \right)$$

$$= -u_{\frac{\pi}{2}}(t) \sin\left(t - \tfrac{\pi}{2}\right)$$

alors

$$\mathcal{L}_f(x) = \frac{x}{x^2+1} + e^{-\frac{\pi}{2}x} \cdot \frac{1}{x} + \frac{1}{x^2+1} e^{-\frac{\pi}{2}x}$$

(13) 8

$$f(t) = \begin{cases} 1 & 0 \le t \le \frac{\pi}{4} \\ \sin 2t & \frac{\pi}{4} < t \le \frac{\pi}{2} \\ 0 & t > \frac{\pi}{2} \end{cases}$$

$$f(t) = u_0 - u_{\frac{\pi}{4}} + \sin 2t \left(u_{\frac{\pi}{4}} - u_{\frac{\pi}{2}} \right)$$

$$\sin 2\left(t - \tfrac{\pi}{4} + \tfrac{\pi}{4}\right) u_{\frac{\pi}{4}} - \sin 2\left(t - \tfrac{\pi}{2} + \tfrac{\pi}{2}\right) u_{\frac{\pi}{2}}$$

$$= \left[\sin 2(t - \tfrac{\pi}{4}) \cos \tfrac{\pi}{2} + \sin \tfrac{\pi}{2} \cos 2(t - \tfrac{\pi}{4})\right] u_{\pi/4}$$

$$- \left[\sin 2(t - \tfrac{\pi}{2}) \cos \pi - \sin \pi \cos 2(t - \tfrac{\pi}{2})\right] u_{\pi/2}$$

$$= \cos 2(t - \tfrac{\pi}{4}) u_{\frac{\pi}{4}} + \sin 2(t - \tfrac{\pi}{2}) u_{\pi/2}$$

$$\mathcal{L}_f(x) = e^{-\frac{\pi}{4}x} \frac{x}{x^2+4} + e^{-\pi/2 x} \frac{2}{x^2+4}$$

$$+ \frac{1}{x}\left(1 - e^{-\pi/4 x}\right)$$