Vzor

Pf. $L^{-1}\left(\frac{1}{(p^2+1)^2}\right)$

$$L\Big(\big(f*g\big)(t)\Big)(x) = F(x)\,G(x)$$

Víme $L(\sin t) = \dfrac{1}{p^2+1}$ $= F(x)$
$= G(x)$

pak

$$\frac{1}{(p^2+1)^2} = F(x)\,G(x) = L(\sin t * \sin t)$$

$$\downarrow$$

$$\int_0^t \sin y \,\sin(t-y)\,dy$$

$$= \frac{1}{2}\sin t - \frac{1}{2}\cos t.$$

Obraz periodické funkce

Př $$f(t) = \begin{cases} 1 & 0 \le t \le 1 \\ -1 & 1 < t < 2 \end{cases}$$

tedy $f(t) = u_0 - 2u_1 + 1u_2 \qquad t \ge 0$

perioda = 2

$$F = \frac{1}{s}(1 - 2e^{-s} + e^{-2s})$$

Věta o periodě:

$$F(s) = \frac{1 - 2e^{-s} + e^{-2s}}{s(1-e^{-2s})} = \frac{(1-e^{-s})^2}{s(1+e^{-s})(1-e^{-s})} = \frac{1-e^{-s}}{s(1+e^{-s})}$$

Př $$f(t) = \begin{cases} \sin t & 0 \le t \le \pi \\ 0 & \pi < t < 2\pi \end{cases}$$

$$f(t) = \sin t \,(u_0(t) - u_\pi(t)) = \sin t \; u_0(t) + \sin(t-\pi) u_\pi(t)$$

tedy $$F = \frac{1}{s^2+1} + \frac{1}{s^2+1} e^{-\pi s}$$

periodicita:

$$F(s) = \frac{1 + e^{-\pi s}}{(s^2+1)(1-e^{-2\pi s})} = \frac{1}{(s^2+1)(1-e^{-\pi s})}$$

Vzor impulsu

Př $F(s) = \frac{1}{s^2} e^{-2s} + \frac{1}{s} e^{-3s}$

$$\frac{1}{s^2} \downarrow t \qquad \frac{1}{s} \downarrow 1$$

tedy $L^{-1}(F)(t) = (t-2) u_2 + 1 \cdot u_3 \qquad t \geq 0$

$$f(t) = \begin{cases} 0 & 0 \leq t \leq 2 \\ t-2 & 2 < t \leq 3 \\ t-1 & t > 3 \end{cases}$$

Př $F(s) = \frac{2}{s(s-1)} e^{-s}$

$$= \frac{-2}{s} + \frac{2}{s-1}$$

tedy $f(t) = -2 \cdot u_1 + 2 e^{t-1} u_1 \qquad t \geq 0$

Př $F(s) = \frac{1}{p+1} \left(e^{-p-1} - e^{-2p-2} \right)$

$$f(t) = e^{-1} e^{-(t-1)} u_1(t) - e^{-2} e^{-(t-2)} u_2(t) \qquad t \geq 0$$

Vzor Laplace

<u>Př</u> $F(x) = \dfrac{x+1}{x^2-x}$

$\to$ 2 póly $0, 1$

$$\operatorname{Res}_0 F(x)\, e^{xt} = -1$$

$$\operatorname{Res}_1 F(x)\, e^{xt} = 2e^t$$

pak pro $t > 0$ máme

$$f = -1 + 2e^t$$

<u>Př</u> $F(x) = \dfrac{1}{(x+1)(x-1)^3(x^2+1)}$

$$\text{póly} \quad \left.\begin{matrix} -1 \\ i \\ -i \end{matrix}\right\} \text{1. st.}$$

$$1 \to \text{3. řádu}$$

$$\operatorname{Res}_{-1} F(x)\, e^{xt} = -\frac{1}{16} e^{-t}$$

$$i \qquad = \frac{1}{8} e^{it}$$

$$-i \qquad = -\frac{1}{8} e^{-it}$$

$$1 \qquad = \frac{2t^2 - 6t + 5}{2} e^t$$

$$f(t) = -\frac{1}{16} e^{-t} + \frac{1}{8} e^{it} - \frac{1}{8} e^{-it} + \frac{1}{2}(2t^2 - 6t + 5) e^t$$

Pf $\quad \sin(t) * t = \int_0^t \sin(y)(t-y)\,dy =$

$$= \int_0^t t\sin y - y\sin y\,dy = [(-\cos y)\,t]_0^t$$

$$- \int_0^t \underset{u}{y}\,\underset{v'}{\sin y}\,dy = -t\cos t + t - [-\cos y\; y]_0^t$$

$$u' = 1 \quad v = -\cos y$$

$$+ \int_0^t -\cos y = -t\cancel{\cos t} + t + t\cancel{\cos t}$$

$$-[\sin y]_0^t = \underline{\underline{t - \sin t}}$$