$$\boxed{y'' + y = k\, u_a(t)} \qquad y(0) = 0 \qquad y'(0) = 1 \qquad a > 0$$

$$s^2 L(y) - s\,y(0) - y'(0) + L(y) = k\,\frac{e^{-as}}{s}$$

$$\Rightarrow \quad L(y) = \frac{1}{s^2+1} + k\,\frac{e^{-as}}{s(s^2+1)}$$

tedy $\quad y = \sin t + k\,u_a(t)(1 - \cos(t-a))$

$$\boxed{y'' + y = \begin{cases} \sin t & 0 \le t \le \pi \\ 0 & t > \pi \end{cases}} \qquad y(0) = y'(0) = 0$$

méně

$$s^2 L(y) + L(y) = \int_0^\pi e^{-st} \sin t \, dt$$

$$= \frac{-e^{-st}}{s^2+1}(s \cdot \sin t + \cos t)\Big|_0^\pi$$

$$= \frac{e^{-\pi s}}{s^2+1} + \frac{1}{s^2+1}$$

$$L(y) = \frac{1}{(s^2+1)^2} + \frac{e^{-\pi s}}{(s^2+1)^2}$$

pak

$$y = \frac{1}{2}(\sin t - t\cos t) + u_\pi(t)\left[\frac{1}{2}(\sin(t-\pi) - (t-\pi)\cos(t-\pi))\right].$$

ODR

DÚ?

$$\boxed{y''' + y'' = e^t + t + 1} \qquad y(0) = y'(0) = y''(0) = 0$$

Laplace:

$$L(y''') + L(y'') = L(e^t) + L(t) + L(1)$$

tedy

$$s^3 L(y) - s^2 y(0) - s y'(0) - y''(0)$$
$$+ s^2 L(y) - s y(0) - y'(0) = \frac{1}{s-1} + \frac{1}{s^2} + \frac{1}{s}$$

tedy

$$s^3 L(y) + s^2 L(y) = \frac{2s^2 - 1}{s^2(s-1)}$$

Soustavy

$$\boxed{\frac{dy}{dt} = -z \qquad \frac{dz}{dt} = y} \qquad y(0) = 1, \quad z(0) = 0$$

$$L(y') = -L(z) \qquad\qquad L(z') = L(y)$$

$$\Rightarrow \quad sL(y) - 1 = -L(z) \qquad sL(z) = L(y)$$

vyřešíme

$$s^2 L(y) - s = -sL(z) = -L(y)$$

$$\rightsquigarrow \quad L(y) = \frac{s}{s^2+1}$$

$$\rightsquigarrow \quad y = \cos t, \qquad z = -y' = \sin t$$

$$\left| \begin{array}{l} y' + z' + y + z = 1 \\ \qquad y' + z = e^t \end{array} \right| \qquad y(0) = -1 \qquad z(0) = 2$$

1\. rce : $s\,L(y) + 1 + s\,L(z) - 2 + L(y) + L(z) = \frac{1}{s}$

$$s\,L(y) + 1 + L(z) = \frac{1}{s-1}$$

$$\rightsquigarrow \quad L(y) = \frac{-s^2 + s + 1}{s(s-1)^2}$$

$$\rightsquigarrow \quad y = 1 - 2e^t + te^t \qquad z = 2e^t - te^t$$

$$\boxed{\phi(t) = t^2 + \int_0^t \phi(y)\sin(t-y)\,dy}$$

$$L(\phi) = \frac{2}{s^3} + L(\phi)\,\frac{1}{s^2+1}$$

$$\rightarrow \quad L(\phi) = \frac{2}{s^5} + \frac{2}{s^3}$$

invers

$$\rightarrow \quad \phi(t) = t^2 + \frac{1}{12}t^4$$

$$\boxed{\int_0^t \varphi(u)\,\varphi(t-u)\,du = 16\sin 4t}$$

$$= \quad \varphi(t) * \varphi(t) = 16\sin 4t$$

$$L(\varphi)\cdot L(\varphi) = \frac{64}{s^2+16}$$

$$\rightarrow \quad L(\varphi) = \frac{\pm 8}{\sqrt{s^2+16}}$$

$$\rightarrow \quad \varphi = L^{-1}\left(\frac{\pm 8}{\sqrt{s^2+16}}\right)$$

$$\boxed{\phi(x) - \lambda \int_0^x e^{x-y}\, \phi(y)\, dy = f(x)}$$

↳ ob. fo

$$\rightarrow \quad \phi(x) - \lambda \; e^x * \phi(x) = f(x)$$

$$L(\phi) - \lambda L(\phi) \frac{1}{s-1} = L(f)$$

$$L(\phi) \left( \frac{s-1-\lambda}{s-1} \right) = L(f)$$

$$L(\phi) = L(f) + \lambda L(f) \frac{1}{s-(1+\lambda)}$$

$$\rightarrow \quad \phi = f + \lambda \; f * e^{(1+\lambda)x}$$

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$$\underline{3\,y(t) - 4\,y(t-1) + y(t-2) = t}$$ $y(t) = 0$ pro $t < 0$

Laplace

$$3\,Ly - 4\,L(y(t-1)) + L(y(t-2)) = \frac{1}{s^2}$$

$\rightarrow$

$$3\,Ly - 4e^{-s}\,Ly + e^{-2s}\,Ly = \frac{1}{s^2}$$

$\rightarrow$

$$Ly = \frac{1}{s^2(3-4e^{-s}+e^{-2s})}$$

$$\boxed{a_{n+2} - 5a_{n+1} + 6a_n = 0}\,, \quad a_0 = 0 \quad a_1 = 1$$

definujme $y(t) = a_n$ $n \leq t < n+1$

$\rightarrow$

$$y(t+2) - 5y(t+1) + 6\,y(t) = 0$$

Laplace

$$e^{2s}Ly(s) - \frac{e^{s}(1-e^{-s})}{s} - 5\,e^{s}Ly(s) + 6Ly(s) = 0$$

pak

$$L(y) = \frac{e^{s}(1-e^{s})}{s(e^{2s}-5e^{s}+6)} = \frac{1-e^{-s}}{s}\left\{\frac{1}{1-3e^{-s}} - \frac{1}{1-2e^{-s}}\right\}$$

$\rightarrow$

$$\underline{\underline{a_n = 3^n - 2^n}}$$