a) $\lim_{n\to\infty} \frac{n}{1-ni} \cdot \frac{1+ni}{1+ni} = \lim_{n\to\infty} \frac{n+n^2 i}{1+n^2}$ $\begin{cases} \operatorname{Re} \lim_{n\to\infty} \frac{n}{1+n^2} = \lim_{n\to\infty} \frac{n^2 \frac{1}{n}}{n^2(\frac{1}{n^2}+1)} = 0 \\ \operatorname{Im} \lim_{n\to\infty} \frac{n^2 i}{1+n^2} \end{cases}$

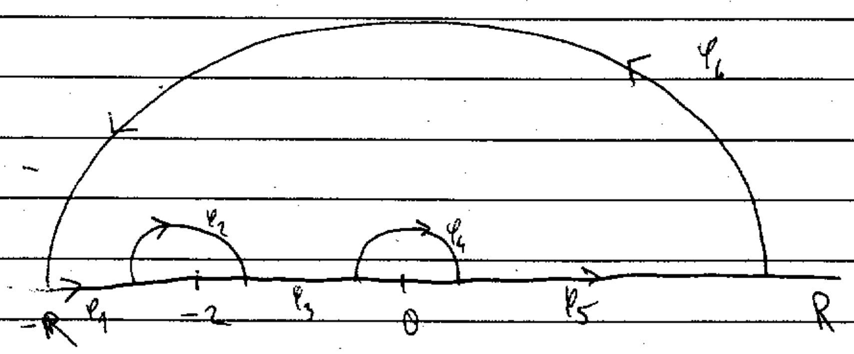
$i + 0 = i$

1. $\overline{\text{CVIKO IX}}$

$I = \int_{-\infty}^{\infty} \frac{(x-3)\sin \pi x}{x(x+2)} = \operatorname{Im} \int_{-\infty}^{\infty} \frac{(x-3)e^{i\pi x}}{x(x+2)} dx$

$rez_0 = \lim_{x\to 0} \frac{(x-3)\,x\,e^{i\pi x}}{x(x+2)} = -\frac{3}{2}$

$rez_{-2} = \lim_{x\to -2} \frac{(x+2)(x-3)e^{i\pi x}}{x(x+2)} = \lim_{x\to -2} \frac{(x-3)e^{i\pi x}}{x} = \frac{5}{2}$



$\int_{\varphi\ldots} = 0$ (cauchy)

$\int_{\varphi} = \int_{\varphi_1} + \int_{\varphi_2} + \int_{\varphi_3} + \int_{\varphi_4} + \int_{\varphi_5} + \int_{\varphi_6}$

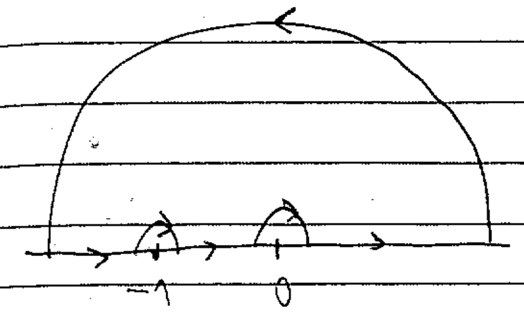
$I = \varphi_1 + \varphi_3 + \varphi_5$ teda $I = +\pi$

$\frac{5}{2} \cdot \frac{3}{2}$ $\int_{\varphi_2} = -\frac{3}{2}$ $\Big\} = \operatorname{Im}(\pi i) = \pi$ (77,8)

$\frac{3}{2}$ $\int_{\varphi_4} = \frac{5}{2}$

$\frac{3}{2}\pi - \frac{5}{2}\pi$ $\int_{\varphi_6} = 0$ (jordan)

2. $\int_{-\infty}^{\infty} \frac{\sin \pi x}{x^2+x} = \operatorname{Im} \int_{-\infty}^{\infty} \frac{e^{i\pi z}}{z^2+z} dz$



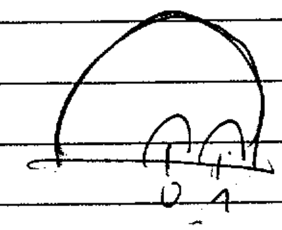
$rez_0 = \lim_{z\to 0} \frac{z e^{i\pi z}}{z(z+1)} = 1$

$rez_{-1} = \lim_{z\to -1} e^{iz} = -e^{-i\pi} = 1$

$(1+1) - \pi i = \operatorname{Im} -2\pi i = -2\pi$

$I = 2\pi$

3. $\int_{-\infty}^{\infty} \frac{\sin \pi x}{x^2-x} dx = \operatorname{Im} \int_{-\infty}^{\infty} \frac{e^{i\pi z}}{z(z-1)} dz$

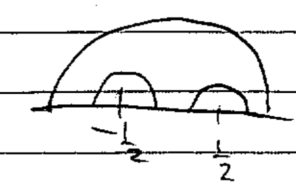


$rez_0 f = \lim_{z\to 0} \frac{e^{i\pi z}\,\cancel{z}}{\cancel{z}(z-1)} = -1$

$rez_1 f = \lim_{z\to 1} \frac{e^{i\pi z}}{z} = e^{i\pi} = -1$

$\int_{\varphi_2} + \int_{\varphi_4} = \operatorname{Im} \pi i + \pi i = 2\pi$

$I = -2\pi$



4. $\int_{-\infty}^{\infty} \frac{(x-8)\cos \pi x}{4x^2-1} dx = \operatorname{Re} \int_{-\infty}^{\infty} \frac{(z-8)\, e^{z\pi i}}{4z^2-1} dz =$

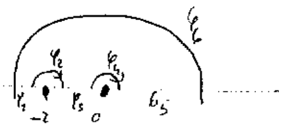
korene $z_1 = \frac{1}{2}$ $z_2 = -\frac{1}{2}$

$rez_{-\frac{1}{2}} = \lim_{x\to -\frac{1}{2}} \frac{(x+\frac{1}{2})(x-8)e^{x\pi i}}{4(x+\frac{1}{2})(x-\frac{1}{2})} = -\frac{17}{8} i$ $\int_{\varphi_2} = -\frac{17}{8}\pi$

$\Big\} -4\pi$ $I = 4\pi$

$rez_{\frac{1}{2}} = \lim_{x\to \frac{1}{2}} \frac{(x-\frac{1}{2})(x-8)e^{x\pi i}}{4(x+\frac{1}{2})(x-\frac{1}{2})} = -\frac{15}{8} i$ $\int_{\varphi_4} = -\frac{15}{8}\pi$

$$\int_{-\infty}^{\infty} \frac{(x-3)\sin \pi x}{x(x+2)}\,dx = \operatorname{Im} \int_{-\infty}^{\infty} \frac{(z-3)\, e^{i\pi x}}{x(x+2)} = -\operatorname{Im}\left(-\pi i\left(\tfrac{5}{2}-\tfrac{3}{2}\right)\right) = \underline{\underline{+\pi}}$$



$$\int_\varphi f\,dz = 0$$

$$res_0 = \lim \frac{(x-3)\, x\, e^{i\pi x}}{x(x+2)} = -\frac{3}{2}$$

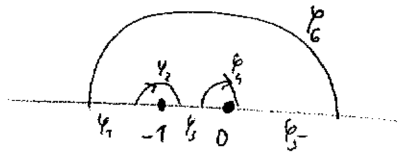
$$res_{-2} = \lim \frac{(x-3)(x+2)\, e^{i\pi x}}{x(x+2)} = \frac{5}{2}$$

$$\int_{\varphi_1+\varphi_3+\varphi_5} \cancel{f} - \pi + 0 = 0$$

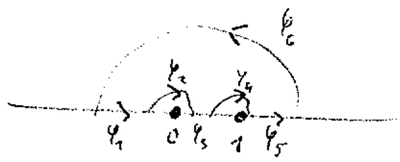
$$\int_{-\infty}^{\infty} \frac{\sin \pi x}{x^2+x} = \operatorname{Im}\int_{-\infty}^{\infty} \frac{e^{i\pi z}}{z(z+1)} = \operatorname{Im}(\pi i\, 2) = 2\pi$$

$$res_0 = \lim_{z\to 0} \frac{z\, e^{i\pi z}}{z(z+1)} = 1$$

$$res_{-1} = \lim_{z\to -1} \frac{(z+1)\, e^{i\pi z}}{z(z+1)} = 1$$



$$\int_{-\infty}^{\infty} \frac{\sin \pi x}{x^2-x}\,dx = \operatorname{Im}\int_{-\infty}^{\infty} \frac{e^{i\pi z}}{z(z-1)}\,dz = \operatorname{Im}(\pi i(-2)) = \underline{\underline{-2\pi}}$$



$$res_0 = \lim_{z\to 0} \frac{z\, e^{i\pi z}}{z(z-1)} = -1$$

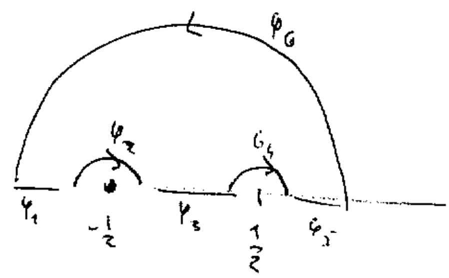
$$res_{+1} = \lim_{z\to +1} \frac{(z-1)\, e^{i\pi z}}{z(z-1)} = -1$$

$$0 = \int_\varphi f\,dz = \int_{\varphi_1+\varphi_3+\varphi_5} + \int_{\varphi_6}^{0} + \pi i + \pi i$$

$$\int_{-\infty}^{\infty} \frac{\cos \pi x\,(x-8)}{4x^2-1}\,dx = \operatorname{Re}\int_{-\infty}^{\infty} \frac{(z-8)\,\cancel{\cos \pi z}\, e^{i\pi z}}{4(z-\frac{1}{2})(z+\frac{1}{2})}\,dz$$

$$res_{\frac{1}{2}} = \lim \frac{(z-8)\,\cancel{\cos}\, e^{i\pi z}(z-\frac{1}{2})}{4(z-\frac{1}{2})(z+\frac{1}{2})}\,dz = -\frac{15}{8}i$$

$$res_{-\frac{1}{2}} = \lim \frac{(z-8)(z+\frac{1}{2})\, e^{i\pi z}}{4(z-\frac{1}{2})(z+\frac{1}{2})}\,dz = -\frac{17}{8}i$$



$$0 = \int_\varphi f\,dz = \int_{\varphi_1+\varphi_3+\varphi_5} + \int_{\varphi_6}^{0} + \left(-\frac{17}{8}i\right)\pi i + \left(-\frac{15}{8}i\right)\pi i$$

$$\longrightarrow \int_{\varphi_1+\varphi_3+\varphi_5} f\,dz = \underline{\underline{4\pi}}$$