Program for Friday seminar on harmonic analysis

In the coming winter semester of the year 2024/2025, lectures at the 'Seminar on harmonic analysis, (Friday, 10.40 - 12.10) will review three chapters from the book

E. Meinrenken: Clifford algebras and Lie theory

The notion of a Weil algebra has a fascinating history. It is based on two short papers written by Henri Cartan in the year 1950 ([3, 4]). A half century later, W. Guillemin and S. Sternberg published a book [2], which describes in detail original motivation of the notion of Weyl algebra coming from algebraic topology and the further important evolution happening during the next 50 years after publication of Cartan's papers.

The series of lectures on these topics at the seminar will start with a summary of first chapters of the book [2] showing the motivation coming from equivariant cohomology of a topological space and desribing an algebraic reformulation of these ideas. It will then continue with a presentation of the contents of Chapter 6 - 8 of the Meinrenken's book.

The contents of the three chapters:

The Chapter 6 contains a discussion of a **Weil algebra** $W(\mathfrak{g})$, which is the tensor product of the symmetric and exterior algebra of the dual $\mathfrak{g}^*$ of a Lie algebra $\mathfrak{g}$. It contains a description of differential spaces and $\mathfrak{g}$-differential spaces. Weil algebra is a universal object among commutative $\mathfrak{g}$-differential algebras with connection. Similarly, non-commutative Weil algebra is a universal object among non-commutative $\mathfrak{g}$-differential algebras with connection. Both algebras have applications to Chern-Weil theory and to transgression.

Chapter 7: Clifford algebra $Cl(V)$ of a Euclidean vector space is a quantization of the exterior algebra $\Lambda(V)$ and the envelopping algebra $\mathcal{U}(\mathfrak{g})$ of a Lie algebra $\mathfrak{g}$ is a quantization of the symmetric algebra $S(\mathfrak{g})$. If $\mathfrak{g}$ is a Lie algebra with a non-degenerate scalar product, then the **quantum Weil algebra** $\mathcal{W}(\mathfrak{g}) = \mathcal{U}(\mathfrak{g}) \otimes Cl(\mathfrak{g})$ is the quantization of the Weil algebra $W(\mathfrak{g})$. The main result in this chapter is the existence of an isomorphism of $\mathfrak{g}$-differential spaces $\mathcal{W}(\mathfrak{g})$ and $W(\mathfrak{g})$, which restricts to a linear isomorphism Φ on basic subcomplexes. The Duflo's theorem is then saying that Φ respects product structures for the case of quadratic Lie algebras.

Chapter 8 describes applications to the case of complex reductive Lie algebras. It contains discussion of the Harish-Chandra projections for the enveloping algebra and its counterpart for Clifford algebra, application to the proof of classical Freudenthal-de Vries "strange formula", and the Gross-Kostant-Ramond-Sternberg theory of multiplets for equal rank Lie subalgebras and its explanation in terms of the cubic Dirac operator, due to Kostant.

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References

[1] E. Meinrenken: Clifford algebras and Lie theory, Springer, Series og modern surveys in mathetmatics, vo; 58, 2013.
[2] W. Guillemin, S. Sternberg, J. Brüning: Supersymmetry and equivariant de Rham theory, Springer, 1999
[3] H. Cartan: Notions d'algebre différentielle; applications aux groupes de Lie et aux variétés oú opére un groupe de Lie, Colloque de topologie, Bruxelles, 1950, pp. 15–27.
[4] H. Cartan: La trangression dans un groupe de Lie et un space fibré principal, Colloque de topologie, Bruxelles, 1950, pp. 57–71.