Some open problems in Banach Space Theory

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Asplund

Problem

X Asplund.

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Does there exists a Lipschitz F bump?

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X Asplund.
Does there exists a Lipschitz F bump?
Does there exists a F bump?

Problem

X with a F bump. Does there exists a Lipschitz F bump?

Problem

$\exists$ X nonreflexive Banach st $NA(X^*)$ linear subspace of X^{**}?

Problem

$X^* \setminus NA(X)$ spaceable. Is X non-reflexive?

Problem [Diestel]

X Banach. RNP $\Leftrightarrow$ KMP ?

Problem [Godefroy]

X Banach. *X* KK*. Is it true that *X* is Asplund?

Problem [Hájek–Talponen' 2013]

Find examples of X with (KK) and not (K).

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Does the long James space $J(\Gamma)$ has (K*)?

LUR renorming

Problem

X separable, $\ell_1 \not\hookrightarrow X$, is X^* $\langle LUR \rangle$?

Problem [Hájek–Talponen' 2013]

X separable, $\ell_1 \not\hookrightarrow X$, is $X^* \langle KK \rangle$?

Problem [Borwein–Fabian' 93]

Prove for all X, $\exists\, f : X \to \mathbb{R}$ convex continuous somewhere Gâteaux non Fréchet without Josefson–Nissenzweig.

Asplund

Problem

X separable such that $\| \cdot \|$ restricted to any closed subspace is somewhere Fréchet. Is X Asplund?

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Remarks on the set of norm-attaining functionals and diffeentiability.
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