

# Lecture Notes in Mathematics

Edited by A. Dold and B. Eckmann

Subseries: Mathematisches Institut der  
Universität Erlangen-Nürnberg

Adviser: H. Bauer



1189

Jaroslav Lukeš  
Jan Malý  
Luděk Zajíček

Fine Topology Methods  
in Real Analysis  
and Potential Theory



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