

$$\text{Te} \quad c_n = \int_{-1/2}^{1/2} r e^{-2\pi i n r} dr = \int_{-1/2}^{1/2} r (\underbrace{\cos 2\pi n r}_{\substack{\uparrow \\ \text{even}}} + i \underbrace{\sin 2\pi n r}_{\substack{\uparrow \\ \text{odd}}}) dr = \text{CVS/2}$$

$$= -2i \int_0^{1/2} r \sin 2\pi n r dr$$

\downarrow $-\frac{\cos 2\pi n r}{2\pi n}$
 per partes

$$= +2i \left[r \frac{\cos 2\pi n r}{2\pi n} \right]_0^{1/2} + 2i \int_0^{1/2} \frac{\cos 2\pi n r}{2\pi n} dr$$

$$= + \frac{(-1)^m i}{2\pi n} + 2i \left[\frac{\sin 2\pi n r}{(2\pi n)^2} \right]_0^{1/2} = \frac{(-1)^{m+1} i}{2\pi n}$$

$$\text{Te} \quad (c_0=0)$$

$$\tilde{w}(t, r) = \sum_{m \in \mathbb{Z}} \underbrace{\frac{\sin 2\pi \ell |m| t}{2\pi \ell |m|}}_{\text{odd } m} \underbrace{\frac{(-1)^m i}{2\pi m}}_{\text{odd } m} (\underbrace{\cos 2\pi n r}_{\text{even}} + i \underbrace{\sin 2\pi n r}_{\text{odd}})$$

$$= \sum_{n=1}^{\infty} \frac{\sin 2\pi \ell |m| t}{2\pi \ell |m|} \frac{(-1)^{m+1}}{\pi m} \sin 2\pi n r$$

$$\Rightarrow v(t, r) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sin 2\pi \ell n t}{2\pi^2 \ell n^2} \frac{\sin 2\pi n r}{r}$$