The Hodgkin-Huxley equations

A patch of the membrane with voltage dependent ion channels is described by the Hodgkin-Huxley equations. We intended to include in the abstract booklet at least one non-trivial neuronal model. This includes all the parameters necessary to reproduce the neuronal firing. Therefore, as an example, the system describing the firing in the avian nucleus magnocellularis cells is shown here:

$$\begin{aligned} C\frac{dV}{dt} &= G_{Na}m_{Na}^2 h_{Na}(V_{Na}-V) + G_K m_K^2 h_K (V_K - V) + G_L(V_L - V) + I, \\ \frac{dj}{dt} &= \frac{j_{\rm ss} - j}{\tau_j} \quad \text{for} \quad j = m_{Na}, h_{Na}, m_K, h_K, \end{aligned} \tag{1}$$

where V, C, I are voltage, capacitance and input current, t is time and G_{Na}, G_K and G_L are maximal **sodium ($\mathbf{Na^+}$)**, **potassium ($\mathbf{K^+}$)** ion channel, and "leakage" **(L)** conductances. V_{Na}, V_K, and V_L are the reversal potentials for the appropriate ions. The second equation represents the system of four equations for the activation and the inactivation of voltage sensitive ion channels. Activation (m) and inactivation (h) variables are for both Na^+ and K^+ ions. They are denoted $j = m_{Na}, h_{Na}, m_K, h_K$. Their time constants are τ_j and steady state voltage dependences $j_{\rm ss}$ are given by the sigmoideal (Boltzmann) curves:

$$j_{\rm ss} = \frac{1}{1 + \exp\left((V_{j,\rm half} - V)/K_j\right)}, \tag{2}$$

where $V_{j,\rm half}$ is the half-activation voltage and K_j is the slope coefficient of the Boltzmann curve. $G_L = 1$ nS, reversal potentials are $V_{Na} = 50$ mV, $V_K = -95$ mV and $V_L = -66$ mV, and the rest of active parameters are in the Table below. (In the contrast to the usual potassium current, the current used here inactivates and thus has both m_K, and h_K variables.) When starting from initial conditions: $V(0) = -66$ mV, $m_{Na}(0) = 0.14,\ h_{Na}(0) = 1,\ m_K(0) = 0,\ h_K(0) = 1$, which are close to steady state values, the system stays at rest.

constant	units	m, Na	h, Na	m, K	h, K
$V_{j,\rm half}$	mV	-40	-45	-54	-50
K_j	mV	3	-3	6.5	-6.5
τ_j	ms	0.05	0.5	0.43	1.2
$G_{\rm ion}$	nS	200	200	120	120

Table: Example parameters of the Hodgkin-Huxley equations