Computing disconnected bifurcation diagrams of partial differential equations

Patrick Farrell
Mathematical Institute, University of Oxford

Computing the distinct solutions u of an equation $f(u, \lambda) = 0$ as a parameter $\lambda \in \mathbb{R}$ is varied is a central task in applied mathematics and engineering. The solutions are captured in a bifurcation diagram, plotting (some functional of) u as a function of λ. In this talk I will present a new algorithm, deflated continuation, for this task.

Deflated continuation has three advantages. First, it is capable of computing disconnected bifurcation diagrams; previous algorithms only aimed to compute that part of the bifurcation diagram continuously connected to the initial data. Second, its implementation is extremely simple: it only requires a minor modification to an existing Newton-based solver. Third, it can scale to very large discretisations if a good preconditioner is available.

Among other problems, we will apply this to a famous singularly perturbed ODE, Carrier's problem. The computations reveal a striking and beautiful bifurcation diagram, with an infinite sequence of alternating pitchfork and fold bifurcations as the singular perturbation parameter tends to zero. The analysis yields a novel and complete taxonomy of the solutions to the problem. We will also apply it to discover previously unknown solutions to equations arising in liquid crystals and quantum mechanics.