Ex. 40: Geometric distribution

ii) a iii) $S = \sum X_i$, U(S) je NNO parametru p

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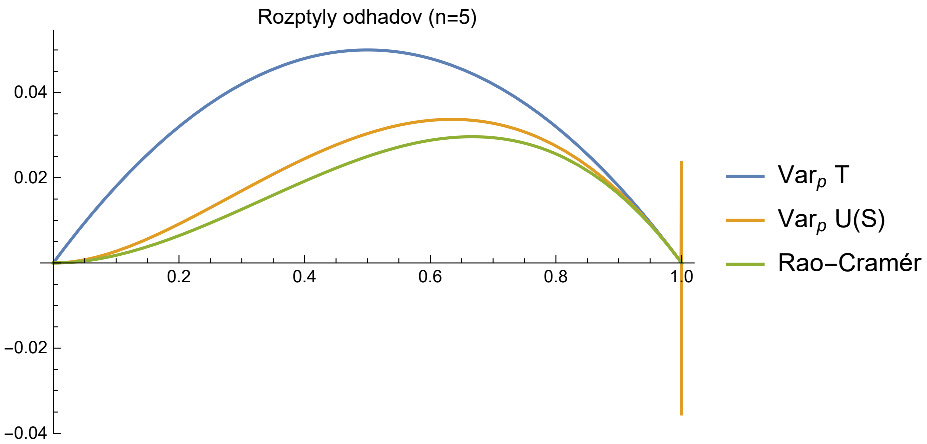
$Assumptions = 1 > p > 0 && n ≥ 1;
u[j_] = PDF[NegativeBinomialDistribution[n, p], j];
Sum[(1 - 1/n)/(1 + j/n - 1/n) u[j], {j, 0, ∞}] (* E U(S) je p *)
Var = Sum[((1 - 1/n)/(1 + j/n - 1/n) - p)^2 u[j], {j, 0, ∞}]
(* Var U(S) je komplikovaná funkcia *)
FI = (-n D[Log[PDF[GeometricDistribution[p], x]], {p, 2}] /.
      x → Mean[GeometricDistribution[p]] // Simplify)^-1
(* Rao-Cramérova medza pre p je p^2(1-p)/n *)
Plot[Evaluate[{p (1 - p)/n, Var, FI} /. n → 5],
 {p, 0, 1}, PlotLabel → "Rozptyly odhadov (n=5)",
 PlotLegends → {"Var_p T", "Var_p U(S)", "Rao-Cramér"}]
```

p

$-p^2 + p^n \,\text{Hypergeometric2F1}[-1 + n, -1 + n, n, 1 - p]$

$$\frac{p^2 - p^3}{n}$$



iv) $S = \sum X_i$, U(S) je NNO parametru p(1-p)

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Sum[(n - 1) s / ((n + s - 1) (n + s - 2)) u[s], {s, 0, ∞}] (* E U(S) je p(1-p) *)
Var = Sum[((n - 1) s / ((n + s - 1) (n + s - 2)) - p (1 - p))^2 u[s], {s, 0, ∞}]
(* Var U(S) je komplikovaná funkcia *)
g[p_] = p (1 - p);
Plot[Evaluate[{(p (1 - p)) (1 - p (1 - p)) / n, Var, D[g[p]]^2 FI} /. n → 5], {p, 0, 1},
 PlotRange → {Automatic, {0, (p (1 - p)) (1 - p (1 - p)) / n /. {n → 5, p → 1/2}}},
 PlotLabel → "Rozptyly odhadov (n=5)",
 PlotLegends → {"Var_p T", "Var_p U(S)", "Rao-Cramér"}]
```

$p - p^2$

$-\frac{1}{n}(-1+p)\left(-n p^2 + n p^3 + p^n \text{HypergeometricPFQ}[\{2, -1+n, -1+n\}, \{1, 1+n\}, 1-p]\right)$