Katedra pravděpodobnosti a matematické statistiky

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Change point - postupné změny

Statistický seminář

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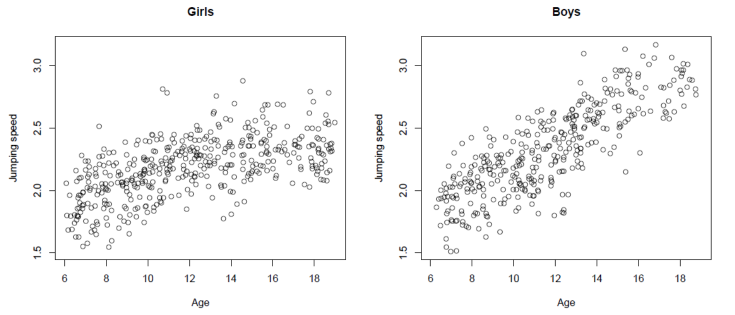
Příklad

- Data zaznamenávají rychlosti odrazu 432 dívek a 364 chlapců ve věku od 6-19 let.
- Cíl: nalézt věk, ve kterém se rychlosti odrazu mezi dívkami a chlapci ve stejné věkové kategorii začínají lišit.



Příklad

Age cat.	girls		boys	
	$\overline{Y_1}$ $(\widehat{\sigma}_1)$	n_1	$\overline{Y_2}$ $(\widehat{\sigma}_2)$	n_2
6–7	1.89 (0.17)	33	1.87 (0.18)	19
7–8	2.00 (0.21)	43	1.98 (0.20)	38
8–9	2.01 (0.21)	33	2.06 (0.21)	38
9–10	2.06 (0.18)	42	2.14 (0.18)	29
10–11	2.19 (0.22)	42	2.17 (0.19)	45
11–12	2.23 (0.15)	30	2.31 (0.23)	37
12–13	2.26 (0.13)	41	2.35 (0.23)	40
13–14	2.30 (0.22)	32	2.53 (0.21)	36
14–15	2.28 (0.23)	31	2.66 (0.19)	20
15–16	2.37 (0.17)	29	2.72 (0.22)	26
16–17	2.33 (0.19)	17	2.83 (0.28)	9
17–18	2.35 (0.18)	25	2.76 (0.16)	13
18–19	2.33 (0.17)	34	2.87 (0.10)	14

Dvojvýběrový t-test a Bonferroni

- Můžeme testovat 13 dvouvýběrových t-testů.
- Využijeme korekce na vícenásobné testování - Bonferonni a Benjamini- Hochberg.
- Zaznamenáváme rozdíl pro chlapce ve věku 13 a výše.
- Zvážíme jiný přístup: Change point.

Dvojvýběrový t-test a Bonferroni

Age cat.	girls		boys		p-values		
	$\overline{Y}_1$ $(\widehat{\sigma}_1)$	n_1	$\overline{Y}_2$ $(\widehat{\sigma}_2)$	n_2	t-test	Bonferroni	BH
6–7	1.89 (0.17)	33	1.87 (0.18)	19	0.780	1.000	0.780
7–8	2.00 (0.21)	43	1.98 (0.20)	38	0.646	1.000	0.763
8–9	2.01 (0.21)	33	2.06 (0.21)	38	0.369	1.000	0.479
9–10	2.06 (0.18)	42	2.14 (0.18)	29	0.081.	1.000	0.117
10–11	2.19 (0.22)	42	2.17 (0.19)	45	0.713	1.000	0.773
11–12	2.23 (0.15)	30	2.31 (0.23)	37	0.062.	0.800	0.100
12–13	2.26 (0.13)	41	2.35 (0.23)	40	0.047*	0.615	0.088.
13–14	2.30 (0.22)	32	2.53 (0.21)	36	0.000***	0.001***	0.000***
14–15	2.28 (0.23)	31	2.66 (0.19)	20	0.000***	0.000***	0.000***
15–16	2.37 (0.17)	29	2.72 (0.22)	26	0.000***	0.000***	0.000***
16–17	2.33 (0.19)	17	2.83 (0.28)	9	0.001***	0.006**	0.001**
17–18	2.35 (0.18)	25	2.76 (0.16)	13	0.000***	0.000***	0.000***
18–19	2.33 (0.17)	34	2.87 (0.10)	14	0.000***	0.000***	0.000***

Předpoklady

- Mějme předpoklady

  $(A1)$ pozorování Y_{jik} jsou získána v čase i,

  $(A2)$ Všechna pozorování jsou nezávislá,

  $(A3)$ $E(\overline{Y_{1i}} - \overline{Y_{2i}}) = \mu + \delta\left(\frac{i-k_0}{n}\right)_+ (i = 1, ..., n),$

  $(A4)$ $Var(Y_{jik}) = \sigma^2_{ji} > 0,$

  kde μ, δ jsou neznámé parametry, $k_0 = n\theta_0$ pro nějaké $\theta_0 \in (0, 1)$, $j = 1, 2$; $i = 1, ..., n$; $k = 1, ..., n_{ji}$.

- Budeme využívat značení $a_+ = \max(a, 0)$.

- Dodatečný předpoklad:
  $(A4^*)$ $\quad Var(\overline{Y_{1i}} - \overline{Y_{2i}}) = \frac{\sigma^2}{m}$, $\sigma^2 > 0$ je neznámý parametr a m může záviset na n.

- Dostáváme LSE odhady :

$$\hat{k_\mu} = \arg \max_{k \in (1,n)} \left[\frac{\{\sum_{i=1}^n (x_{ik} - \overline{x_k})(\overline{Y_{1i}} - \overline{Y_{2i}})\}^2}{\sum_{i=1}^n (x_{ik} - \overline{x_k})^2} \right],$$

$$\hat{\delta_\mu} = \frac{\sum_{i=1}^n (x_{ik} - \overline{x_k})(\overline{Y_{1i}} - \overline{Y_{2i}})}{\sum_{i=1}^n (x_{i\hat{k}} - \overline{x_{\hat{k}}})^2},$$

$$\hat{\mu} = \frac{1}{n} \sum_{i=1}^n (\overline{Y_{1i}} - \overline{Y_{2i}}) - \hat{\delta_\mu} \overline{x_{\hat{k}}}.$$

- Za předpokladu, že $\mu = 0$ máme

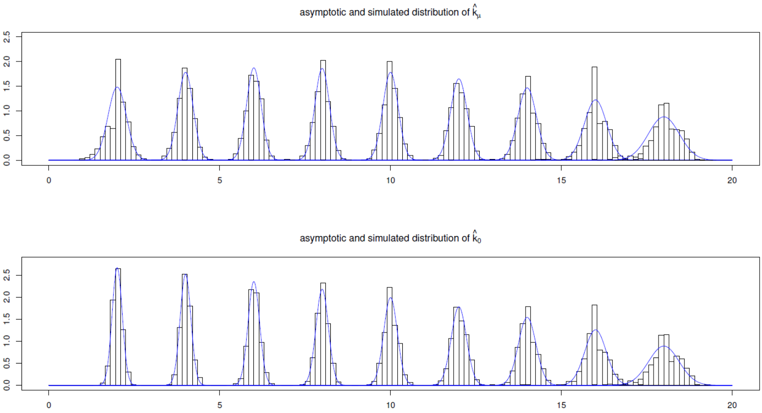
$$\hat{k}_0 = \arg\max_{k \in (1,n)} \left[\frac{\{\sum_{i=1}^{n} x_{ik}(\overline{Y_{1i}} - \overline{Y_{2i}})\}^2}{\sum_{i=1}^{n} x_{ik}^2} \right],$$

$$\hat{\delta}_0 = \frac{\sum_{i=1}^{n} x_{i\hat{k}}(\overline{Y_{1i}} - \overline{Y_{2i}})}{\sum_{i=1}^{n} x_{i\hat{k}}^2},$$

- Platí následující

$$\sqrt{nm}\frac{\delta}{\sigma}\left[\frac{\theta_0(1-\theta_0)}{1+3\theta_0}\right]^{\frac{1}{2}}\frac{\hat{k}_\mu - k_0}{n} \xrightarrow[n\to\infty]{d} N(0,1),$$

$$\sqrt{nm}\frac{(1-\theta_0)^{\frac{3}{2}}}{\sigma}\left(\frac{1+3\theta_0}{12}\right)^{\frac{1}{2}}(\hat{\delta}_\mu - \delta) \xrightarrow[n\to\infty]{d} N(0,1).$$



Obrázek: Hustoty asymptotického rozdělení a histogramy 1000 simulovaných hodnot $\hat{k}_\mu$ a $\hat{k}_0$ pro $n = 20, \sigma^2 = 1, m = 20, \mu = 0, \delta = 1, k_0 \in \{2, 4, ...18\}$.

- $\tau^2 = Var(\overline{Y_{1i}} - \overline{Y_{2i}}) = \frac{\sigma^2_{1i}}{n_{1i}} + \frac{\sigma^2_{2i}}{n_{2i}},$

$$\hat{k}_0(\tau^2) = \arg\max_{k\in(1,n)} \left[\frac{\{\sum_{i=1}^n x_{ik}(\overline{Y_{1i}} - \overline{Y_{2i}})/\tau_i^2\}^2}{\sum_{i=1}^n x_{ik}^2/\tau_i^2} \right],$$

- $\hat{\tau^2} = \frac{\hat{\sigma_{1i}}^2}{n_{1i}} + \frac{\hat{\sigma_{2i}}^2}{n_{2i}},$

$$\hat{k}_0(\hat{\tau^2}) = \arg\max_{k\in(1,n)} \left[\frac{\{\sum_{i=1}^n x_{ik}(\overline{Y_{1i}} - \overline{Y_{2i}})/\hat{\tau_i^2}\}^2}{\sum_{i=1}^n x_{ik}^2/\hat{\tau_i^2}} \right] = \arg\max_{k\in(1,n)} T_{2,\hat{\tau^2}}(k).$$

- Odhadneme parametry δ a k_0
- Spočteme vyrovnané hodnoty $\hat{D}_i = \hat{\delta}_0 \left(\frac{i - \hat{k}}{n} \right)_+ (i = 1, ..., n)$
- Pro $b = 1$ do $b = B$:
  - Generujeme $D_i^* = \hat{D}_i + \hat{\tau}_i \epsilon_i^* \quad (i = 1, ..., n), \quad \epsilon_i^* \sim N(0, 1)$ jsou nezávislé
  - Spočteme change-point odhad $\hat{k}_b^*$ z bootstrapového výběru $D_1^*, ..., D_n^*$
- Spočteme empirický kvantil q_α^* z $\hat{k}_1^* - \hat{k}, ..., \hat{k}_B^* - \hat{k}$ pro $\alpha \in (0, 1)$

- Zvolíme n, change point $\theta_0 = \frac{k_0}{n}$, n_{ji} a $\sigma^2_{j,i}$ pro $j \in \{1, 2\}$ a $i = 1, ..., n$.
- Vypočteme rozptyl $\tau^2 = \frac{\sigma^2_{1i}}{n_{1i}} + \frac{\sigma^2_{2i}}{n_{2i}}$.
- Pro $s = 1$ do $s = S$:
  - Pro $i = 1$ do $i = n$:
    - Generujeme $D_i = \overline{Y_{1i}} - \overline{Y_{2i}}$ (normální rozdělení)
    - Generujeme $\hat{\tau_i}^2$ (χ^2 rozdělení)
  - Spočteme $\hat{k_0}^{(s)}$ odhad change point
  - Spočteme 95% interval spolehlivosti pro k_0 pomocí algoritmu Bootstrap
- Spočteme odchylku, MSE a pravděpodobnost pokrytí

Simulace

Homoskedastický případ

		θ_0	$\widehat{\sigma}^2_{\text{pooled}}$				$\widehat{\sigma}^2_{ji}$			
			$\widehat{k}_\mu$	$\widehat{k}_0$	$\widehat{k}_\mu^{\text{corr}}$	$\widehat{k}_0(\widehat{\tau}^2)$	$\widehat{k}_\mu$	$\widehat{k}_0$	$\widehat{k}_\mu^{\text{corr}}$	$\widehat{k}_0(\widehat{\tau}^2)$
$n = 10$	$n_{ji} = 10$	0.1	88.8	95.2	93.3	94.0	89.5	93.9	93.7	94.5
		0.2	92.4	95.9	92.8	94.9	91.7	94.8	94.3	95.6
		0.4	94.1	92.3	92.9	91.9	95.5	92.4	93.3	92.0
		0.6	92.8	93.2	92.6	92.4	93.9	89.9	92.2	90.2
		0.8	90.4	90.5	90.0	90.8	89.6	87.7	89.2	89.1
		0.9	78.1	78.3	79.2	78.4	80.1	74.8	76.4	76.0
	$n_{ji} = 20$	0.1	93.9	92.0	94.4	92.1	95.1	92.8	94.6	93.0
		0.2	96.3	92.8	95.6	92.4	95.4	92.7	95.8	94.7
		0.4	93.5	92.0	92.3	91.1	92.7	91.6	92.0	90.8
		0.6	87.6	90.1	90.1	89.8	89.3	87.1	88.9	88.4
		0.8	88.8	89.0	89.7	87.8	89.9	86.9	87.2	88.4
		0.9	72.1	70.4	74.9	70.1	72.4	70.9	72.6	70.3
$n = 20$	$n_{ji} = 10$	0.1	96.5	94.3	94.0	94.9	94.9	93.5	95.8	93.1
		0.2	97.1	94.1	95.0	93.7	96.9	93.0	95.0	93.6
		0.4	94.0	93.7	93.8	93.9	94.4	92.1	93.0	92.8
		0.6	93.2	90.9	92.7	92.7	91.9	92.1	91.7	91.8
		0.8	94.8	95.6	94.3	95.3	93.8	94.5	92.5	93.7
		0.9	84.1	84.3	84.8	84.0	83.1	81.8	84.4	80.9
	$n_{ji} = 20$	0.1	97.3	95.0	94.4	94.9	97.0	93.5	93.4	95.3
		0.2	95.1	94.3	94.1	94.3	93.5	93.9	94.1	94.0
		0.4	93.0	93.1	93.1	92.9	93.6	93.6	93.1	94.7
		0.6	91.9	90.7	92.8	92.8	91.7	93.6	92.0	91.4
		0.8	93.2	91.9	91.3	90.7	91.8	92.5	91.5	89.3
		0.9	79.5	81.4	83.4	79.3	82.3	80.4	82.4	82.4

	Nr. of observations (n_{ji})		
	$n_{ji} = m$	$m\{1 + 3I(i \text{ odd})\}/2$	$m\{1 + 3I(i > n/2)\}/2$
σ_{ji} constant ($\sigma_{ji} = \sigma$)		H01	H02
$\sigma_{ji} = \sigma(1 + 2I(i > k_0))$	H10	H11	H12
$\sigma_{ji} = \sigma(1 + 2I(i \text{ even}))$	H20	H21	H22

		$n = 10$								$n = 20$			
	θ_0	$\widehat{\sigma}^2_{\text{pooled}}$				$\widehat{\sigma}^2_{ji}$				$\widehat{\sigma}^2_{ji}$			
		$\widehat{k}_\mu$	$\widehat{k}_0$	$\widehat{k}_\mu^{\text{corr}}$	$\widehat{k}_0(\widehat{\tau}^2)$	$\widehat{k}_\mu$	$\widehat{k}_0$	$\widehat{k}_\mu^{\text{corr}}$	$\widehat{k}_0(\widehat{\tau}^2)$	$\widehat{k}_\mu$	$\widehat{k}_0$	$\widehat{k}_\mu^{\text{corr}}$	$\widehat{k}_0(\widehat{\tau}^2)$
H01	0.1	65.1	66.8	58.0	67.9	88.1	92.5	92.5	93.2	97.5	93.1	95.1	93.8
	0.4	69.9	68.9	67.7	70.7	96.3	94.7	95.7	92.2	94.1	93.3	94.4	95.1
	0.8	74.7	71.2	72.2	68.5	88.5	86.5	86.7	88.4	95.9	94.9	95.2	91.4
	0.9	84.1	82.8	77.9	80.5	78.6	77.4	78.8	78.1	77.5	80.4	80.8	78.9
H02	0.1	60.5	63.2	50.6	65.6	83.7	94.2	92.5	93.9	95.3	93.0	95.0	93.1
	0.4	65.9	65.4	63.9	69.6	90.6	88.4	91.0	93.0	93.6	92.5	92.8	93.2
	0.8	69.4	69.5	74.1	72.9	90.4	89.2	91.0	86.3	89.6	88.4	90.7	90.4
	0.9	80.0	78.2	77.2	83.7	76.8	71.9	75.6	75.8	82.6	84.2	82.3	81.9
H10	0.1	90.0	93.9	92.4	94.0	88.8	94.6	93.4	92.8	92.6	94.0	93.1	95.3
	0.4	97.1	99.6	99.7	99.7	92.7	91.5	94.3	91.1	94.6	94.9	94.9	93.3
	0.8	89.0	93.6	93.3	92.2	91.6	92.5	90.7	89.0	95.3	91.4	94.8	90.5
	0.9	94.1	87.5	87.5	68.5	92.4	88.8	86.1	73.4	87.5	86.6	89.1	82.3
H11	0.1	65.6	65.4	55.6	69.9	89.9	93.1	91.8	93.1	90.4	95.5	91.7	94.2
	0.4	71.0	67.2	67.4	70.8	92.4	94.8	93.2	91.9	93.7	92.4	93.1	93.3
	0.8	79.0	78.7	76.1	71.0	92.2	84.3	90.7	89.8	96.8	95.9	96.4	89.3
	0.9	95.5	92.5	84.6	73.1	93.1	90.3	87.5	66.2	88.4	86.8	86.3	80.1
H12	0.1	58.9	63.9	49.9	64.0	88.3	95.6	92.0	94.2	88.8	94.4	92.4	93.1
	0.4	70.3	68.3	65.3	69.5	90.0	87.1	88.6	91.5	94.5	93.0	94.7	92.9
	0.8	79.6	77.5	78.8	72.2	91.1	90.1	92.7	87.9	93.9	88.5	91.3	88.8
	0.9	93.2	88.3	81.7	78.0	91.3	85.2	85.7	74.3	88.6	87.3	90.9	82.2
H20	0.1	80.9	92.4	91.0	99.4	82.1	91.5	88.0	98.5	97.0	97.2	98.9	99.3
	0.4	93.8	94.6	93.2	99.8	93.0	89.6	89.9	93.7	97.4	95.4	96.6	99.2
	0.8	78.0	75.5	79.1	74.1	78.2	77.4	76.2	73.8	93.2	90.6	91.1	94.1
	0.9	90.6	86.2	84.8	78.3	90.0	86.5	83.3	77.5	79.0	78.4	83.7	74.4
H21	0.1	58.8	63.8	48.8	66.9	76.6	88.7	80.7	94.5	87.8	87.4	87.0	93.3
	0.4	60.6	62.8	61.9	65.2	83.9	82.0	83.3	88.4	85.0	85.4	84.2	90.8
	0.8	73.3	70.7	68.0	69.3	79.9	79.4	76.9	79.8	84.7	84.8	84.3	88.1
	0.9	93.3	88.7	81.8	82.5	87.6	85.4	81.3	78.9	81.1	81.9	79.7	77.4
H22	0.1	49.8	58.4	39.4	69.1	61.6	84.4	73.7	94.8	79.0	83.2	80.4	93.0
	0.4	57.1	61.2	56.0	72.5	74.1	68.2	75.6	90.2	78.0	81.9	81.0	93.3
	0.8	72.6	67.5	72.1	67.6	82.6	79.0	74.9	73.5	76.7	75.4	75.5	89.4
	0.9	92.2	86.6	79.2	79.9	90.3	83.7	83.2	82.1	83.9	78.7	81.9	76.8

Zaokrouhlování

- Mějme i- tou kategorii tzn. věk od $i - i + 1$ a $k_0 \in (0, n)$.
- Víme, že

$$\mathrm{E}(\overline{Y_{1i}} - \overline{Y_{2i}}) = 0 \quad \text{pro} \quad i \leq \lfloor k_0 \rfloor,$$

$$\mathrm{E}(\overline{Y_{1i}} - \overline{Y_{2i}}) = \frac{\delta(i - k_0)}{n} \quad \text{pro} \quad i \geq \lceil k_0 \rceil.$$

- Označíme si $i_0 = \lfloor k_0 \rfloor$ a $d_0 = k_0 - i_0$ a spočteme si

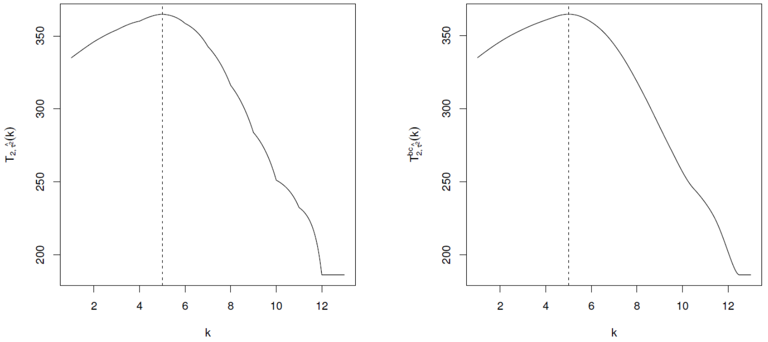
$$\mathrm{E}(\overline{Y_{1i_0}} - \overline{Y_{2i_0}}) = \mathrm{E}\left[\left(\frac{\delta(X - k_0)}{n}\right)_+\right].$$

			$\widehat{k}_0$			$\widehat{k}_0^{bc}$		
		θ_0	MSE	bias	coverage	MSE	bias	coverage
$n = 10$	$n_{ji} \equiv 10$	0.20	0.124	0.003	93.6%	0.112	−0.010	94.6%
		0.22	0.113	−0.001	95.4%	0.113	−0.016	95.3%
		0.25	0.125	−0.018	91.5%	0.123	−0.010	92.9%
		0.28	0.122	0.012	91.2%	0.133	0.011	90.3%
		0.30	0.116	0.008	93.9%	0.131	0.001	95.0%
		0.70	0.432	−0.109	92.7%	0.365	−0.041	93.3%
		0.72	0.466	−0.099	94.0%	0.498	−0.065	91.7%
		0.75	0.678	−0.151	89.6%	0.726	−0.138	88.4%
		0.78	0.936	−0.226	86.5%	0.971	−0.123	91.5%
		0.80	1.160	−0.263	91.3%	1.255	−0.224	91.8%
	$n_{ji} \equiv 20$	0.20	0.053	−0.011	94.1%	0.052	0.002	93.2%
		0.22	0.054	−0.013	95.4%	0.050	0.003	98.4%
		0.25	0.053	−0.011	95.8%	0.060	−0.017	95.4%
		0.28	0.059	0.001	92.8%	0.054	−0.009	95.2%
		0.30	0.063	0.000	92.6%	0.060	0.011	92.8%
		0.70	0.177	−0.056	86.9%	0.173	0.004	96.3%
		0.72	0.194	−0.073	94.1%	0.191	−0.024	95.6%
		0.75	0.246	−0.072	94.6%	0.230	−0.052	91.7%
		0.78	0.319	−0.116	88.1%	0.302	−0.034	93.2%
		0.80	0.399	−0.097	88.0%	0.506	−0.092	96.9%
$n = 20$	$n_{ji} \equiv 10$	0.20	0.054	−0.005	93.6%	0.050	−0.004	95.7%
		0.22	0.056	−0.005	94.5%	0.057	−0.003	95.3%
		0.25	0.056	−0.016	94.6%	0.050	0.003	95.4%
		0.28	0.061	0.002	94.5%	0.055	−0.009	94.6%
		0.30	0.063	−0.004	93.1%	0.060	−0.012	93.9%
		0.70	0.150	−0.040	93.7%	0.158	0.001	94.7%
		0.72	0.175	−0.035	93.9%	0.163	−0.033	94.5%
		0.75	0.200	−0.051	93.3%	0.178	−0.023	96.7%
		0.78	0.220	−0.043	94.0%	0.229	−0.026	90.8%
		0.80	0.263	−0.050	94.7%	0.276	−0.022	93.8%
	$n_{ji} \equiv 20$	0.20	0.027	−0.006	94.0%	0.026	−0.004	93.3%
		0.22	0.026	−0.008	94.3%	0.024	−0.010	98.0%
		0.25	0.030	−0.006	93.3%	0.029	0.005	93.6%
		0.28	0.026	0.005	95.8%	0.030	0.001	95.9%
		0.30	0.033	−0.015	93.9%	0.031	−0.004	94.1%
		0.70	0.078	−0.024	90.7%	0.072	−0.000	94.1%
		0.72	0.089	−0.032	96.7%	0.075	−0.001	95.9%
		0.75	0.093	−0.037	90.6%	0.095	−0.006	93.4%
		0.78	0.112	−0.046	95.3%	0.103	−0.012	94.3%
		0.80	0.114	−0.040	90.0%	0.111	−0.012	95.7%

Značení

Index (k)	Label	Meaning	Interpretation	$\overline{Y_1}$ ($\widehat{\sigma}_1$)	$\overline{Y_2}$ ($\widehat{\sigma}_2$)
1	6	6–7 years	∼ 6.5 years	1.89 (0.17)	1.87 (0.18)
2	7	7–8 years	∼ 7.5 years	2.00 (0.21)	1.98 (0.20)
⋮	⋮	⋮	⋮	⋮	⋮
13	18	18–19 years	∼ 18.5 years	2.33 (0.17)	2.87 (0.10)

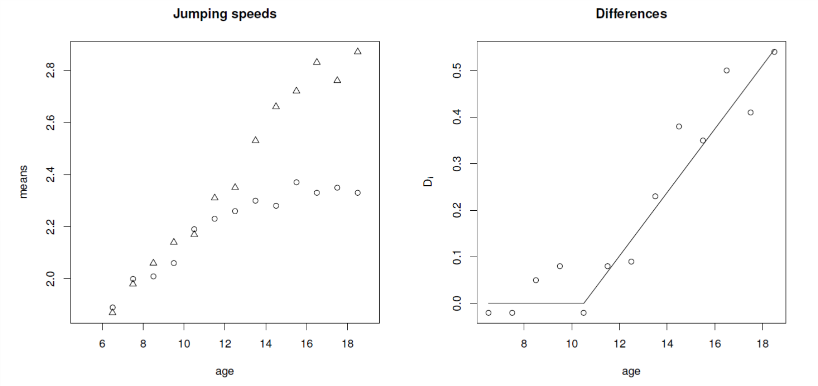
Funkce



Obrázek: Funkce $T_{2,\hat{\tau}^2}(k)$ a $T^{bc}_{2,\hat{\tau}^2}(k)$ pro data rychlostí odrazu.

Age cat.	girls $\overline{Y}_1$ $(\widehat{\sigma}_1)$	girls n_1	boys $\overline{Y}_2$ $(\widehat{\sigma}_2)$	boys n_2	p-values t-test	p-values Bonferroni	p-values BH	p-values $\widehat{k}_0(\widehat{\tau}^2)$	p-values $\widehat{k}_0^{bc}(\widehat{\tau}^2)$	Age
6–7	1.89 (0.17)	33	1.87 (0.18)	19	0.780	1.000	0.780	1.000	1.000	6
7–8	2.00 (0.21)	43	1.98 (0.20)	38	0.646	1.000	0.763	1.000	1.000	7
8–9	2.01 (0.21)	33	2.06 (0.21)	38	0.369	1.000	0.479	1.000	1.000	8
9–10	2.06 (0.18)	42	2.14 (0.18)	29	0.081.	1.000	0.117	0.999	0.997	9
10–11	2.19 (0.22)	42	2.17 (0.19)	45	0.713	1.000	0.773	0.861	0.846	10
11–12	2.23 (0.15)	30	2.31 (0.23)	37	0.062.	0.800	0.100	0.113	0.117	11
12–13	2.26 (0.13)	41	2.35 (0.23)	40	0.047*	0.615	0.088.	0.003**	0.003**	12
13–14	2.30 (0.22)	32	2.53 (0.21)	36	0.000***	0.001***	0.000***	0.000***	0.000***	13
14–15	2.28 (0.23)	31	2.66 (0.19)	20	0.000***	0.000***	0.000***	0.000***	0.000***	14
15–16	2.37 (0.17)	29	2.72 (0.22)	26	0.000***	0.000***	0.000***	0.000***	0.000***	15
16–17	2.33 (0.19)	17	2.83 (0.28)	9	0.001***	0.006**	0.001**	0.000***	0.000***	16
17–18	2.35 (0.18)	25	2.76 (0.16)	13	0.000***	0.000***	0.000***	0.000***	0.000***	17
18–19	2.33 (0.17)	34	2.87 (0.10)	14	0.000***	0.000***	0.000***	0.000***	0.000***	18

Ověření


Jumping speeds
means
2.0
2.2
2.4
2.6
2.8
6
8
10
12
14
16
18
age
Differences
D_i
0.0
0.1
0.2
0.3
0.4
0.5
8
10
12
14
16
18
age

Děkujeme za pozornost!