NMST545 – Mathematical statistics 4

Homework assignments

- The solution of these assignments is a part of the final examination. It will be counted towards the final grade.
- You are required to send your commented R code together with a pdf of a report at least 24 hours before the start of the exam by e-mail to nagy@karlin.mff.cuni.cz. Scanned hand-written reports are fine, but must be readable.
- Before the resampling procedure in each part of your solution, set **set.seed(AAA)**, where **AAA** stands for the number of your student identity card. From your report, it must be clear how the resampling is done for each particular task.
- In your report, it is expected that you provide a brief description of the method used, its assumptions, and comment on the results. You should also provide a numerical value of the estimators, confidence intervals, test statistics... Do not also forget to **specify the assumed model** and give the formulas so that it is clear how the result is calculated.
- Use 5 % as the level (prescribed probability of type I error) of the tests and 95 % as the coverage of the confidence intervals.

Task 1

Load the dataset Iq-en.RData (available here).

(load("Iq-en.RData"))

For n = 111 children, the dataset contains their gender (Iq\$Gender), IQ (Iq\$iq), and the average marks in the seventh (Iq\$mark7) and the eighth grade (Iq\$mark8).

- (i) Recall that the skewness of a distribution is defined as $\gamma = \mathsf{E}\left(\frac{X-\mu}{\sigma}\right)^3$, where $\mu = \mathsf{E} X$ and $\sigma^2 = \mathsf{var} X$. Using an appropriate bootstrap procedure, find a confidence interval for the skewness of the average marks in the eighth grade of students.
- (ii) Perform a bootstrap test of the null hypothesis that the average mark in the seventh grade follows a shifted beta distribution, given by the law of 1 + 3Z. Here, $Z \sim B(\alpha, \beta)$ is beta-distributed for some $\alpha > 0$ and $\beta > 0$.
- (iii) Denote by ρ the correlation coefficient of the average marks in the seventh and the eighth grade. Devise a bootstrap procedure to test the null hypothesis $H_0: \rho = 0.925$ against $H_1: \rho \neq 0.925$.

Task 2

Demonstrate the use of a resampling/bootstrap principle in a problem of your choice. You can use the dataset lq-en.RData, any other dataset considered in the class, or a (possibly simulated) dataset of your own. If you use your own dataset, please attach it together with your solution. Make sure that your solution is not identical to those given on **the website** of the course.