$\log X_i \sim N(\mu, \sigma^2)$

(65) $X_1, \dots, X_n$ i.i.d as $f(x;\mu,\sigma^2) = \frac{1}{\boxed{x} \sqrt{2\pi}\,\sigma} \exp\left\{-\frac{(\log x - \mu)^2}{2\sigma^2}\right\} \mathbb{1}\{x>0\}$

$$\ell_n(\mu,\sigma^2) = \log\left(\prod_{i=1}^{n} f(X_i;\mu,\sigma^2)\right)$$

$$= \sum_{i=1}^{n} \log\left(\frac{1}{X_i\sqrt{2\pi}\sqrt{\sigma^2}} \exp\left\{-\frac{(\log X_i - \mu)^2}{2\sigma^2}\right\}\right)$$

$$= \left(-\frac{n}{2}\log\sigma^2\right) - \left(\frac{1}{2\sigma^2}\sum_{i=1}^{n}(\log X_i - \mu)^2\right) + C$$

$$\frac{\partial \ell_n(\mu,\sigma^2)}{\partial\mu} = -\frac{1}{2\sigma^2}\sum_{i=1}^{n}(\log X_i - \mu)\cdot(-2) \overset{!}{=} 0$$

$$\frac{\partial \ell_n(\mu,\sigma^2)}{\partial\sigma^2} = -\frac{n}{2\sigma^2} + \frac{1}{2(\sigma^2)^2}\sum_{i=1}^{n}(\log X_i - \mu)^2 \overset{!}{=} 0$$

(i)
$$\hat{\mu}_n = \frac{1}{n}\sum_{i=1}^{n}\log X_i$$
$$\hat{\sigma}_n^2 = \frac{1}{n}\sum_{i=1}^{n}(\log X_i - \hat{\mu}_n)^2$$

$$I(\mu,\sigma^2) = \begin{pmatrix} \frac{1}{\sigma^2} & 0 \\ 0 & \frac{1}{2\sigma^2} \end{pmatrix} \quad \frac{1}{2\sigma^4}$$

(ii) $\leadsto$ $$\sqrt{n}\left[\begin{pmatrix}\hat{\mu}_n \\ \hat{\sigma}_n^2\end{pmatrix} - \begin{pmatrix}\mu \\ \sigma^2\end{pmatrix}\right] \xrightarrow[n\to\infty]{d} N_2\left(\begin{pmatrix}0\\0\end{pmatrix}, \begin{pmatrix}\sigma^2 & 0 \\ 0 & 2\sigma^2\end{pmatrix}\right)$$

$2\sigma^4$

$$\underset{\sim}{\theta} = \begin{pmatrix} \mu \\ \sigma^2 \end{pmatrix} : \left\{ \underset{\sim}{\tilde{\theta}} \in \mathbb{R} \times \mathbb{R}_+ : n\left(\hat{\underset{\sim}{\theta}}_n - \underset{\sim}{\tilde{\theta}}\right)^T \hat{J}_{\cancel{x}} \left(\hat{\underset{\sim}{\theta}}_n - \tilde{\theta}\right) \le \chi^2_2(1-\alpha) \right\}$$

$n$

ODHAD FIM V $X_1$

$$\leadsto \hat{J}_{\cancel{x}} = \begin{pmatrix} \frac{1}{\hat{\sigma}_n^2} & 0 \\ 0 & \frac{1}{2\hat{\sigma}_n^2} \end{pmatrix} \qquad \frac{1}{2\hat{\sigma}_n^4}$$

$$\left\{ \begin{pmatrix} \tilde{\mu} \\ \tilde{\sigma}^2 \end{pmatrix} : n\left(\hat{\mu}_n - \tilde{\mu}, \hat{\sigma}_n^2 - \tilde{\sigma}^2\right) \begin{pmatrix} \frac{1}{\hat{\sigma}_n^2} & 0 \\ 0 & \frac{1}{2\hat{\sigma}_n^2} \end{pmatrix} \begin{pmatrix} \hat{\mu}_n - \tilde{\mu} \\ \hat{\sigma}_n^2 - \tilde{\sigma}^2 \end{pmatrix} \le \chi^2_2(1-\alpha) \right\}$$

$$= \left\{ \begin{pmatrix} \tilde{\mu} \\ \tilde{\sigma}^2 \end{pmatrix} \in \mathbb{R} \times \mathbb{R}_+ : \frac{n(\hat{\mu}_n - \tilde{\mu})^2}{\hat{\sigma}_n^2} + \frac{n(\hat{\sigma}_n^2 - \tilde{\sigma}^2)^2}{2\hat{\sigma}_n^2} \le \chi^2_2(1-\alpha) \right\}$$

$\hat{\sigma}_n^4$

(iv) $$\sqrt{n}\left(\hat{\mu}_n - \mu\right) \xrightarrow[n\to\infty]{d} N(0, \sigma^2)$$

$$\left(\hat{\mu}_n - u_{1-\alpha} \frac{\hat{\sigma}_n}{\sqrt{n}} ; \infty\right) \leftarrow \text{ASYMPT. IS}$$

$Y_i = \log X_i \sim N(\mu, \sigma^2)$

MS1: $\left(\hat{\mu}_n - t_{n-1}(1-\alpha)\frac{S_n^{(Y)}}{\sqrt{n}}, \infty\right)$,

kde $S_n^{(Y)} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(Y_i - \bar{Y}_n)^2}$