(79) $\underset{\sim}{X} = (X_1, X_2, X_3)^T \sim Mult_3(n, (\pi_1, \pi_2, \pi_3))$ — AA, Aa, aa

H_0: $\begin{pmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \end{pmatrix} = \begin{pmatrix} \vartheta_x^2 \\ 2\vartheta_x(1-\vartheta_x) \\ (1-\vartheta_x)^2 \end{pmatrix}$ pro nějaké $\vartheta_x \in (0,1)$

H_1: NEPLATÍ H_0

$$\ell_n(\underset{\sim}{\pi}) = \log\left(\frac{n!}{x_1! x_2! x_3!} \prod_{i=1}^{3} \pi_i^{x_i}\right) = \sum_{i=1}^{3} x_i \log \pi_i + c$$

$c = \log\left(\frac{n!}{x_1! x_2! x_3!}\right)$

ZA H_0:

$$\tilde{\ell}_n(\vartheta) = \log\left([\vartheta^2]^{x_1} [2\vartheta(1-\vartheta)]^{x_2} [(1-\vartheta)^2]^{x_3}\right)$$

$$LR_n = 2\left(\sup_{\pi} \ell_n(\pi) - \sup_{\vartheta} \tilde{\ell}_n(\vartheta)\right)$$

$$= 2\left(\ell_n(\hat{\pi}_n) - \tilde{\ell}_n(\tilde{\vartheta}_n)\right)$$

$\rightsquigarrow$ MLO BEZ OMEZENÍ: $\hat{\pi}_n = \left(\frac{x_1}{n}, \frac{x_2}{n}, \frac{x_3}{n}\right)^T$

MLO ZA H_0:

$$\tilde{\ell}_n(\vartheta) = 2x_1 \log \vartheta + x_2 \log[2\vartheta(1-\vartheta)] + 2x_3 \log(1-\vartheta)$$

$$= (2x_1 + x_2)\log\vartheta + (x_2 + 2x_3)\log(1-\vartheta) + x_2 \log 2$$

$$\frac{\partial \tilde{\ell}_n(\theta)}{\partial \theta} = \frac{2X_1 + X_2}{\theta} - \frac{X_2 + 2X_3}{1-\theta} \stackrel{!}{=} 0$$

$$2X_1 + X_2 - \theta(2X_1 + X_2) - \theta(X_2 + 2X_3) = 0$$

$$\tilde{\theta}_n = \frac{2X_1 + X_2}{2(X_1 + X_2 + X_3)} = \frac{2X_1 + X_2}{2n}$$

$$LR_n = 2\left(\ell_n(\hat{\pi}_n) - \tilde{\ell}_n(\tilde{\theta}_n)\right) =$$

$$= 2\left(\sum_{k=1}^{3} X_k \log\left(\hat{\pi}_{nk}\right) - \sum_{k=1}^{3} X_k \log\left(\tilde{\pi}_{nk}\right)\right)$$

$$= 2\sum_{k=1}^{K} X_k \log\left(\frac{\hat{\pi}_{nk}}{\tilde{\pi}_{nk}}\right)$$

$\tilde{\pi}_n = (\tilde{\pi}_{n1}, \dots, \tilde{\pi}_{nK})^T$

$$\tilde{\ell}_n(\tilde{\theta}_n) = \ell_n\left(\tilde{\theta}_n^2,\ 2\tilde{\theta}_n(1-\tilde{\theta}_n),\ (1-\tilde{\theta}_n)^2\right)$$

$$= \ell_n(\hat{\pi}_n) = \sum_{k=1}^{K} X_k \log\left(\hat{\pi}_{nk}\right)$$

$$\text{zum } H_0 \iff LR_n \geq \chi^2_1(1-\alpha)$$

$$\dim(\Theta) = 2,\ \dim(\Theta_0) = 1$$

MST: ≈

(82) $\underset{\sim}{X} \sim Mult_{16}(n, \underset{\sim}{\pi})$, kde

$\underset{\sim}{\pi} = (\pi_{11}, \pi_{12}, \dots, \pi_{43}, \pi_{44})^T$

(ii) $H_0: \pi_{ij} = \pi_{i+} \pi_{+j}$, kde $\pi_{11}, \dots, \pi_{44}$

$\pi_{+1}, \dots, \pi_{+4}$

$$\pi_{i+} = \sum_{j=1}^{J} \pi_{ij}, \quad \pi_{+j} = \sum_{i=1}^{I} \pi_{ij}$$

$$l_n(\underset{\sim}{\pi}) = \log\left(n! \prod_{i=1}^{4} \prod_{j=1}^{4} \frac{1}{x_{ij}!} \pi_{ij}^{x_{ij}} \right) =$$

$$= \sum_{i=1}^{4} \sum_{j=1}^{4} x_{ij} \log \pi_{ij} + C$$

$\hat{\underset{\sim}{\pi}}_n = (\hat{\pi}_{11}, \dots, \hat{\pi}_{44})^T$

MVO BEZ OMEZENÍ: $\hat{\pi}_{ij} = \frac{x_{ij}}{n}$

MVO ZA H_0: $\tilde{\pi}_{ij} = \frac{x_{i+}}{n} \frac{x_{+j}}{n}$

$$LR_n = 2\left(l_n(\hat{\underset{\sim}{\pi}}_n) - l_n(\tilde{\underset{\sim}{\pi}}_n) \right)$$

$$= 2\left[\sum_{i} \sum_{j} x_{ij} \log \hat{\pi}_{ij} - \sum_{i} \sum_{j} x_{ij} \log \tilde{\pi}_{ij} \right]$$

$$= 2 \sum_{i=1}^{4} \sum_{j=1}^{4} x_{ij} \log\left(\frac{\hat{\pi}_{ij}}{\tilde{\pi}_{ij}} \right) \geq \chi^2_9(1-\alpha)$$

$\dim(\Theta) = 15$, $\dim(\Theta_0) = 6$ — za platnosti H_0

(ii) $\tilde{\pi}_{ij} = \frac{X_{ij} + X_{ji}}{2n} = \frac{\hat{\pi}_{ij} + \hat{\pi}_{ji}}{2}$

$H_0 : \pi_{ij} = \pi_{ji} \quad \forall i, j$