1. IDENTIFICATION OF PARAMETERS

(A) $Y_i = \overset{0}{\beta_0} + \overset{1}{\beta_1} X_i + \varepsilon_i \, , \quad \varepsilon_i \perp X_i \, ,$

$$X_i \sim U(0,1) \, , \quad \varepsilon_i \sim N(0,1)$$

$$\mathbb{E}[Y_i | X_i] = \beta_0 + \beta_1 X_i$$

$$F^{-1}_{Y_i|X_i}(\tau) = \beta_0 + \beta_1 X_i + F^{-1}_{\varepsilon}(\tau)$$

$$\rightsquigarrow \beta_0(\tau) = \beta_0 + F^{-1}_{\varepsilon}(\tau)$$

$$\beta_1(\tau) = \beta_1$$

FOR $\underset{\sim}{\hat{\beta}}(\tau) = \begin{pmatrix} \hat{\beta}_0(\tau) \\ \hat{\beta}_1(\tau) \end{pmatrix}$ APPROX. HOLDS:

$$\frac{1}{n} \sum_{i=1}^{n} \underset{\sim}{X_i} \, \psi_\tau \left(\overbrace{Y_i - \underset{\sim}{X_i}^T \underset{\sim}{\hat{\beta}}(\tau)}^{\hat{\varepsilon}_i} \right) \doteq \begin{pmatrix} 0 \\ 0 \end{pmatrix},$$

where $\underset{\sim}{X_i} = \begin{pmatrix} 1 \\ X_i \end{pmatrix}$,

$$\psi_\tau(x) = \tau \, \mathbb{1}\{x > 0\} - (1-\tau) \, \mathbb{1}\{x < 0\}$$

$$\rightsquigarrow \sum_{i=1}^{n} \psi_\tau(\hat{\varepsilon}_i) = \tau \sum_{i=1}^{n} \mathbb{1}\{\hat{\varepsilon}_i > 0\} - (1-\tau) \sum_{i=1}^{n} \mathbb{1}\{\hat{\varepsilon}_i < 0\} \doteq 0$$

(B) $$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i, \quad \varepsilon_i \perp X_i,$$
$$X_i \sim U(0,1), \quad \varepsilon_i \sim Exp(1)$$

$$\mathbb{E}[Y_i \mid X_i] = \beta_0 + \beta_1 X_i + \mathbb{E}\varepsilon_i =$$
$$= \underbrace{(\beta_0 + 1)}_{\beta_0^{LS}} + \underset{\substack{\uparrow \\ \beta_1^{LS}}}{\beta_1} X_i$$

$$F^{-1}_{Y_i|X_i}(\tau) = \beta_0 + \beta_1 X_i + \underbrace{F^{-1}_{\varepsilon}(\tau)}_{-\log(1-\tau)}$$

$$\beta_0(\tau) = \beta_0 - \log(1-\tau), \quad \beta_1(\tau) = \beta_1$$

(C) $$Y_i = \beta_0 + \beta_1 X_i + \sigma(X_i)\varepsilon_i, \quad \varepsilon_i \perp X_i$$
$$X_i \sim U(0,1), \quad \varepsilon_i \sim N(0,1), \quad \sigma(X_i) = e^{X_i}$$

$$\mathbb{E}[Y_i \mid X_i] = \beta_0 + \beta_1 X_i + \sigma(X_i)\,\mathbb{E}\varepsilon_i$$
$$= \beta_0 + \beta_1 X_i$$

$$F^{-1}_{Y_i|X_i}(\tau) = \beta_0 + \beta_1 X_i + e^{X_i} F^{-1}_{\varepsilon}(\tau)$$

$$\begin{pmatrix} \beta_0(\tau) \\ \beta_1(\tau) \end{pmatrix} : \quad \begin{array}{l} \mathbb{E}\,\psi_\tau\big(Y_i - \beta_0(\tau) - \beta_1(\tau) X_i\big) = 0 \\ \mathbb{E}\,X_i\,\psi_\tau\big(Y_i - \beta_0(\tau) - \beta_1(\tau) X_i\big) = 0 \end{array}$$

① $Y_i = \beta_0 + \beta_1 X_i + e^{X_i} \varepsilon_i, \quad \varepsilon_i \perp X_i$

$$X_i \sim U(0,1), \quad \varepsilon_i \sim \mathrm{Exp}(1)$$

$$\mathbb{E}[Y_i \mid X_i] = \beta_0 + \beta_1 X_i + e^{X_i} \mathbb{E}\varepsilon_i$$

$$= \beta_0 + \beta_1 X_i + e^{X_i}$$

$$\begin{pmatrix} \beta_0^{LS} \\ \beta_1^{LS} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & \mathbb{E}X_i \\ \mathbb{E}X_i & \mathbb{E}X_i^2 \end{pmatrix}}_{\mathbb{E}\, \underset{\sim}{X_i} \underset{\sim}{X_i}^T}^{-1} \underbrace{\begin{pmatrix} \mathbb{E}Y_i \\ \mathbb{E}Y_i X_i \end{pmatrix}}_{\mathbb{E}\, \underset{\sim}{X_i} Y_i}$$

$\boxed{\text{BIRTHWEIGHT}}$

(A) $\underline{\text{BWEIGHT} \sim \beta_0 + \beta_1 \mathbb{1}\{\text{MAGE} \geq 30\}}$

$$\hat{\beta}_1(\tau) = \underset{\uparrow}{\hat{G}^{-1}_{n_2}}(\tau) - \underset{\uparrow}{\hat{F}^{-1}_{n_1}}(\tau)$$

MAGE $\geq$ 30 MAGE < 30

anova.rq(fit)

CONST.

$$H_0: \ \beta_1(\tau) = \beta_1 \quad \forall \tau \in (0,1)$$

(B) $\underline{\text{BWEIGHT} \sim \beta_0 + \beta_1 (\text{MAGE} - \text{MEAN}(\text{MAGE}))}$

i.i.d. $\quad Y_i = \underset{\sim}{X}_i^T \underset{\sim}{\beta} + \varepsilon_i, \quad \varepsilon_i \perp \underset{\sim}{X}_i$

$$\rightsquigarrow \widehat{\text{avar}}\left(\hat{\underset{\sim}{\beta}}(\tau)\right) = \left(\sum_{i=1} \underset{\sim}{X}_i \underset{\sim}{X}_i^T\right)^{-1} \tau(1-\tau) \left[\hat{s}_n(\tau)\right]^2$$

WHERE $\hat{s}_n(\tau) = \widehat{\left(\dfrac{1}{f_\varepsilon(F_\varepsilon^{-1}(\tau))}\right)}$ $\leftarrow$ SPARSITY FUNCTION AT τ

COURSE NOTES: $\hat{s}_n(\tau) = \dfrac{\hat{F}_\varepsilon^{-1}(\tau + h_n) - \hat{F}_\varepsilon^{-1}(\tau - h_n)}{2 h_n}$

SUMMARY.RQ(..., se = "i.i.d")

LAD: $\hat{F}_\varepsilon^{-1}(\varepsilon_j) \approx \tau_j, \quad \tau_j \in \{\lfloor n(\tau - h_n)\rfloor, \ldots, \lfloor n(\tau + h_n)\rfloor\}$

$\hat{s}_n(\tau) = \hat{\beta}_1$

(c) $$\text{BWEIGHT} \sim \beta_0 + \beta_1 (\text{MAGE} - \text{MEAN}(\text{MAGE})) + \beta_2 (\text{BLENGTH} - \text{MEAN}(\text{BLENGTH}))$$

ANOVA.RQ (, SUBMODEL, MODEL) $\leftarrow$ FOR GIVEN τ
OR FOR A SET OF τ