1. MCAR

$$\underset{\sim}{X}_1, \dots, \underset{\sim}{X}_n \text{ i.i.d } N_2\left(\begin{pmatrix}0\\0\end{pmatrix}, \begin{pmatrix}1 & \frac{1}{2} \\ \frac{1}{2} & 1\end{pmatrix}\right)$$

$$\begin{array}{c|c|c} X_{1,1}, \dots, X_{1000,1} & NA, \dots, NA & X_{1501,1}, \dots, X_{2000,1} \\ \hline \underbrace{X_{1,2}, \dots, X_{1000,2}}_{CCA} & X_{1001,2}, \dots, X_{1500,2} & NA, \dots, NA \end{array}$$

CI FOR μ_1: $\left(\hat{\mu}_1 \mp t_{df}(1-\alpha/2)\sqrt{\frac{\hat{\sigma}_{11}^2}{n}}\right)$

IMPUTATION: $\hat{X}_{1000+i,1} = \hat{\alpha}_0 + \hat{\alpha}_1 X_{1000+i,2}$

ESTIM. FROM $\underset{\sim}{X}_1, \dots, \underset{\sim}{X}_{1000}$

MULT. IMPUTATION: $\hat{X}_{1000+i,1} = \hat{\alpha}_0 + \hat{\alpha}_1 X_{1000+i,2} + \hat{\sigma}\varepsilon_i$,

WHERE $\hat{\sigma}^2 = \frac{1}{1000-2}\sum_{i=1}^{\cancel{n}\;1000}\left(X_{i,1} - \hat{\alpha}_0 - \hat{\alpha}_1 X_{i,2}\right)^2$,

$\varepsilon_1, \dots, \varepsilon_{500}$ i.i.d from $N(0,1)$

OBSERVED LIKELIHOOD:

$(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \sigma_{12})$

$$l_{obs}(\underset{\sim}{\theta}) = \sum_{i=1}^{1000}\log\left(f_{X_1,X_2}(X_{i1}, X_{i2}; \underset{\sim}{\theta})\right) + \sum_{i=1001}^{1500}\log\left(f_{X_2}(X_{i2}; \mu_2, \sigma_2^2)\right) + \sum_{i=1501}^{2000}\log\left(f_{X_1}(X_{i1}; \mu_1, \sigma_1^2)\right)$$

EM-ALGORITHM:

- in $l^c_{\sim}$ one needs to calculate:

$$E_{\hat{\theta}^{(k)}}\left[X_{1000+i,1} \mid X_{1000+i,2}\right] \;\&\; E_{\hat{\theta}^{(k)}}\left[X^2_{1000+i,1} \mid X_{1000+i,2}\right]$$

$$\hat{\underset{\sim}{\theta}}^{(k)} = \left(\hat{\mu}_1^{(k)}, \hat{\mu}_2^{(k)}, \hat{\sigma}_1^{2(k)}, \hat{\sigma}_2^{2(k)}, \hat{\sigma}_{12}^{(k)}\right)^T$$

2. MAR

$\underset{\sim}{X}_1, \dots, \underset{\sim}{X}_n \text{ i.i.d } N_2\left(\begin{pmatrix}0\\0\end{pmatrix}, \begin{pmatrix}1 & \frac{1}{2}\\ \frac{1}{2} & 1\end{pmatrix}\right)$

$X_{i,1}$ ALWAYS OBSERVED

$$P(X_{i2} \text{ OBSERVED} \mid X_{i1}) = \frac{e^{4X_{i1}}}{1+e^{4X_{i1}}}$$

REWEIGHTING

$$\hat{\pi}_i = \frac{e^{\hat{\beta}_0 + \hat{\beta}_1 X_{i1}}}{1+e^{\hat{\beta}_0 + \hat{\beta}_1 X_{i1}}},$$

WHERE $(\hat{\beta}_0, \hat{\beta}_1)$ COME FROM THE LOGISTIC.

REGRESSION OF „X_{i2} OBSERVED" $\sim X_{i1}$

$\boxed{\text{3. MNAR}}$ $\underset{\sim}{X_1}, \ldots, \underset{\sim}{X_n}$ i.i.d $N_2\left(\begin{pmatrix}0\\0\end{pmatrix}, \begin{pmatrix}1 & 1/2\\ 1/2 & 1\end{pmatrix}\right)$

X_{i1} ALWAYS OBSERVED

X_{i2} OBSERVED IF $\dfrac{\exp(X_{i1}-X_{i2})}{1+\exp(X_{i1}-X_{i2})} \geq \dfrac{1}{2}$

4. LINEAR REGR. WITH MISS. VALUES IN THE COVAR.

MODEL: $Y_i = \underset{\underset{0}{\uparrow}}{\beta_0} + \underset{\underset{0}{\uparrow}}{\beta_1} X_{i1} + \underset{\underset{1}{\uparrow}}{\beta_2} X_{i2} + \varepsilon_i$

$$\varepsilon_i \perp (X_{i1}, X_{i2})^T \quad \& \quad \varepsilon_i \sim N(0,1)$$

$$\& \begin{pmatrix} X_{i1} \\ X_{i2} \end{pmatrix} \sim N_2\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & 1/2 \\ 1/2 & 1 \end{pmatrix} \right)$$

Y_i, X_{i1} ALWAYS OBSERVED

X_{i2} OBSERVED IF $\dfrac{\exp(X_{i1}+Y_i)}{1+\exp(X_{i1}+Y_i)} \geq \dfrac{1}{2}$

$\boxed{\text{DIRECT LIKELIHOOD}}$

→ NOTE THAT: $\begin{pmatrix} X_{i1} \\ X_{i2} \\ Y_i \end{pmatrix} \sim N_3(\ldots, \mathbb{V})$

$\operatorname{cov}\left(\begin{pmatrix} X_{i1} \\ X_{i2} \end{pmatrix}, Y_i\right) = \mathbb{V}[1:2, 3]$

$$\begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} = \Sigma_{X}^{-1} \, \Sigma_{XY}$$

$\operatorname{var}\left(\begin{pmatrix} X_{i1} \\ X_{i2} \end{pmatrix}\right) = \mathbb{V}[1:2, 1:2]$