$\boxed{\text{1. FAITHFUL DATASET}}$ $X_1, \ldots, X_n$

HISTOGRAMS

$x_0 < x_1 < x_2 < \ldots$, WHERE $x_1 = x_0 + \underbrace{b_n}_{\text{WIDTH}}$

START: x_0

$\vdots$

$x_l = x_{l-1} + b_n$

FOR $x \in (x_j, x_{j+1}]$: $\quad f_n^{(H)}(x) = \frac{1}{n b_n} \sum_{i=1}^{n} \mathbb{1}\{X_i \in (x_j, x_{j+1}]\}$

KERNEL ESTIMATES

$$\hat{f}_n(x) = \frac{1}{n h_n} \sum_{i=1}^{n} K\left(\frac{x - X_i}{h_n}\right)$$

K: A DENSITY

$K(x) = \mathbb{1}\{x \in (-\frac{1}{2}, \frac{1}{2})\}$ ← UNIFORM, RECTANGULAR

$h_n = \frac{b_n}{2}$

density(...): ← $\int_{-\infty}^{\infty} u^2 K(u)\, du = 1$

$K(x) = \frac{1}{2\sqrt{3}} \mathbb{1}\{|x| < \sqrt{3}\}$ ... RECTANG.

$K(x) = \frac{3}{4\sqrt{5}} \left(1 - \frac{x^2}{5}\right)^2 \mathbb{1}\{|x| < \sqrt{5}\}$ ... EPANECH.

$K(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$ ... GAUSSIAN

2. NORMAL MIXTURE

$X_1, \dots, X_n$ i.i.d FROM $\underset{0.5}{w}\, N(\underset{0}{\mu_1}, \underset{1}{\sigma_1^2}) + (1-w)\, N(\underset{4}{\mu_2}, \underset{4}{\sigma_2^2})$

$$K(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

bw.nrd(...) .... $h_n = \dfrac{1.06 \min\{S_n, \widetilde{IQR}_n\}}{n^{1/5}}$

bw.nrd0(...) .... $h_n = \dfrac{0.9 \min\{S_n, \widetilde{IQR}_n\}}{n^{1/5}}$

bw.ucv(...) ... $h_n = \underset{h_n > 0}{\operatorname{argmin}} \mathcal{L}(h_n)$,

WHERE $\mathcal{L}(h_n) = \int [\hat{f}_n(x)]^2 dx - \frac{2}{n} \sum_{i=1}^{n} \hat{f}_{n,i}(X_i)$

$\boxed{\text{3. SIMULATION STUDY}}$

$X_1, \dots, X_n$ i.i.d FROM F

A) $N(0,1)$

$1000 \cdot MISE(\hat{f}_n) \approx \int \mathbb{E}\left[\hat{f}_n(x) - f(x)\right]^2 dx \cdot 1000$

B) $LN(\mu, \sigma^2)$

C) $\frac{1}{2} N(0,1) + \frac{1}{2} N(4,4)$