1. MOTORCYCLE DATA

$$\begin{pmatrix} X_1 \\ Y_1 \end{pmatrix}, \dots, \begin{pmatrix} X_n \\ Y_n \end{pmatrix}$$

X_n ← TIME AFTER THE IMPACT

Y_1 ↑ ACCELERATION

A. GLOBAL POLYNOMIALS

$$\hat{m}_{GP}(x) = \sum_{j=0}^{p} \hat{\beta}_j \cdot x^j ,$$

WHERE $(\hat{\beta}_0, \dots, \hat{\beta}_p) = \underset{b_0, \dots, b_p}{\operatorname{argmin}} \sum_{i=1}^{n} \left(Y_i - b_0 - \dots - b_p X_i^p \right)^2$

B. NW AND LL ESTIMATOR

NADARAYA - WATSON :

- LET $K(x) = \frac{1}{2} \mathbb{1}\{|x| \leq 1\}$

$$\rightsquigarrow \hat{m}_{NW}(x) = \frac{\sum_{i=1}^{n} Y_i \, K\left(\frac{X_i - x}{h_n}\right)}{\sum_{i=1}^{n} K\left(\frac{X_i - x}{h_n}\right)} =$$

$$= \frac{\sum_{i=1}^{n} Y_i \, \mathbb{1}\{|X_i - x| \leq h_n\}}{\sum_{i=1}^{n} \mathbb{1}\{|X_i - x| \leq h_n\}}$$

LOCAL LINEAR

$$\hat{m}_{LL}(x) = \hat{\beta}_0(x)$$

WHERE

$$\left(\hat{\beta}_0(x), \hat{\beta}_1(x)\right) = \underset{b_0, b_1}{\operatorname{argmin}} \sum_{i=1}^{n} \left[Y_i - b_0 - b_1(X_i - x)\right]^2 \cdot \mathbb{1}\{|X_i - x| \leq h_n\}$$

$$\hat{m}_{LL}(x) = \hat{\beta}_0(x) + \hat{\beta}_1(x) \cdot (y - x)\Big|_{y=x}$$

EPANECHNIKOV KERNEL

EpaK $\quad K(x) = \frac{3}{4}(1 - x^2)\mathbb{1}\{|x| \leq 1\}$

$$\rightsquigarrow \quad K\left(\frac{X_i - x}{h_n}\right) \text{ INSTEAD OF } \mathbb{1}\{|X_i - x| \leq h_n\}$$

CV CHOICE OF BANDWIDTH

$$h_n^{(CV)} = \underset{h_n > 0}{\operatorname{argmin}} \frac{1}{n} \sum_{i=1}^{n} \left[Y_i - \hat{m}_{LL}^{(-i)}(X_i)\right]^2$$

RULE OF THUMB

$$h_n^{(ROT)} = n^{-1/5} \left[\frac{R(K)\,\tilde{\sigma}^2 \int w_0(x)\,dx}{\mu_{2K}^2 \frac{1}{n} \sum_{i=1}^{n} \left[\tilde{m}''(X_i)\right]^2 w_0(X_i)}\right]^{1/5}$$

$$w_0(x) = \mathbb{1}\{x \in (0, 60)\}$$

NW vs LL.

$$\hat{m}_{NW}(x) = \sum_{i=1}^{n} Y_i \, w_{ni}^{(NW)}(x)$$

$$\hat{m}_{LL}(x) = \sum_{i=1}^{n} Y_i \, w_{ni}^{(LL)}(x)$$

C. LOCAL POLYNOMIAL ESTIMATION

$$\hat{m}_p(x) = \hat{\beta}_0(x), \quad \text{WHERE}$$

$$\left(\hat{\beta}_0(x), \ldots, \hat{\beta}_p(x)\right) = \underset{b_0, \ldots, b_p}{\operatorname{argmin}} \sum_{i=1}^{n} \left[Y_i - b_0 - \ldots - b_p (X_i - x)^p\right]^2 \cdot K\left(\frac{X_i - x}{h_n}\right)$$

D. LOWESS,

- ROBUSTIFIED LOCAL LINEAR FIT WITH h_n CHOSEN BY k-NN METHOD, WHERE $k = \lfloor nf \rfloor$

DEFAULT $f = \frac{2}{3}$

E. CONDITIONAL VARIANCE ESTIMATION

- AIM AT ESTIMATING $\sigma^2(x) = \mathrm{var}(Y|X=x)$

DIRECT:

$$\hat{\sigma}_n^2(x) = \sum_{i=1}^{n} w_{ni}(x)\, Y_i^2 - \left[\hat{m}_p(x)\right]^2$$

MODIFIED

$$\tilde{\sigma}_n^2(x) = \sum_{i=1}^{n} w_{ni}(x) \left(Y_i - \hat{m}_p(X_i)\right)^2$$