# University Center for Mathematical Modeling, Applied Analysis and Computational Mathematics

Semester Seminar, virtual (via Zoom), Monday June 7, 9:30-15:00

## Schedule

| Time | Speaker | Title |
|---|---|---|
| Morning I | | (chair: Luboš Pick) Opening at 9:30 |
| 9:30 | *Opening* | |
| 9:35 | Marek Cúth | Characterization of (semi-)Eberlein compacta using retractional skeletons |
| 9:55 | Gianmarco Sperone | Unrestricted deformations of thin elastic structures interacting with fluids |
| 10:15 | Scott Congreve | Scott Congreve: *hp*-version Recovered Finite Element Methods |
| 10:35 | *Short break* | (10 minutes) |
| Morning II | | (chair: Vít Dolejší) Starting at 10:45 |
| 10:45 | Giovanni Gravina | Relaxation of functionals with bulk and surface energy terms |
| 11:05 | Malte Kampschulte | Friction from a dissipation point of view |
| 11:25 | Sebastian Schwarzacher | Navier-Stokes-Fourier fluids interacting with elastic shells |
| Afternoon I | | (chair: Ondřej Souček) Starting at 13:00 |
| 13:00 | Ondřej Chrenko | Planet migration in a disk of gas and pebbles |
| 13:20 | Josef Hanuš | Differences in physical properties of large main-belt asteroids |
| 13:40 | *Short break* | (10 minutes) |
| Afternoon II | | (chair: Vít Průša) Starting at 13:50 |
| 13:50 | Karel Tůma | Light-activated shape memory polymers: stent simulation |
| 14:10 | Michal Pavelka | Lack-of-fit reduction: From GENERIC to GENERIC |
| 14:30 | *Closing discussion* | |

## Abstracts

**Scott Congreve: *hp*-version Recovered Finite Element Methods.** A *recovered finite element method* is a Galerkin finite element method constructed via (conforming or classical nonconforming) recovery operators over element-wise discontinuous Galerkin finite spaces. Essentially, we define the finite element method, with some suitable stabilisation term, in terms of the *recovery operator* $\mathcal{E} : V_h \mapsto \tilde{V}_h \cap H_0^1(\Omega)$ mapping a *discontinuous* piecewise polynomial $V_h$ over a triangulation onto *continuous* piecewise polynomial space $\tilde{V}_h \cap H_0^1(\Omega)$ over the same, or finer, triangulation, with the addition of a stabilisation term. The resulting method is defined over an element-wise discontinuous Galerkin finite element space, resulting in a discontinuous solution $u_h$. However, we also simultaneously produces a conforming finite element approximation $\mathcal{E}(u_h)$. These resulting methods have several attractive feature over classical finite element methods and discontinuous Galerkin approaches; notably, conformity of the method is not hard-wired into the finite element space allowing considerable flexibility in choice of the recovery operator, the finite element space $\mathcal{E}(V_h)$, and the stabilisation.

Existing work considers a uniform polynomial degree for the discontinuous and conforming finite element spaces over the triangulation. In this talk, we consider the extension of the recovered finite element method to a discontinuous space consisting of different polynomial degrees on different elements of the triangulation. This method allows for the retro-fitting of existing conforming finite element discretisations with the power of $p$-adaptivity with minimal computational overhead.

This is a joint work with Tristan Pryer.

**Marek Cúth: Characterization of (semi-)Eberlein compacta using retractional skeletons.** Eberlein compact is any compact space which is homeomorphic to a weakly compact set in a Banach space. I will briefly talk about Eberlein compacta, their possible generalizations, examples and briefly mention some our recent results in the area. The talk is based on a recent joint preprint with C. Correa and J. Somaglia available at arxiv.org.

**Giovanni Gravina: Relaxation of functionals with bulk and surface energy terms.** The minimization of energy functionals has a wide range of applications, both in pure and applied disciplines, where the existence of minimizes is routinely proved by means of the so-called "direct method in the Calculus of Variations". This, in turn, relies on showing that the energy under consideration is lower semicontinuous. If this property fails, valuable insight may still be gained by characterizing the lower semicontinuous envelop of the energy, referred to as the relaxed energy.

Motivated by problems in the van der Waals-Cahn-Hilliard theory of liquid-liquid phase transitions, and by some classical examples due to Modica, in this talk, we will study the lower semicontinuity of energy functionals with bulk and surface terms. In particular, special emphasis will be given to uncovering the relationship between the roughness of the domain and the relaxation procedure.

**Ondřej Chrenko: Planet migration in a disk of gas and pebbles.** Planets form in protoplanetary disks and they inevitably migrate (change their orbital distance from the central star) due to gravitational planet-disk interactions. Since gas is more abundant than solids in protoplanetary disks (roughly by a factor of 100), it is generally thought that planetary migration is dominated by the gravity of gas. Recent studies, however, suggest that small solid particles (dust or pebbles) mixed with the gas can develop a strongly asymmetric distribution with respect to an embedded planet. The gravitational torque exerted onto the planet by solid particles then becomes substantial and it can even outperform the influence of gas. In this talk, I will introduce my ongoing study that explores interactions of low-mass planets with a disk of gas and pebbles. Gas and pebbles are modeled as separate fluids coupled by a linear aerodynamic drag term. I will present the governing hydrodynamic equations and outline the challenges in their numerical solution. The aim of my research is to find links between the diverse outcomes of planet migration and the observed diversity of extrasolar planets.

**Josef Hanuš: Differences in physical properties of large main-belt asteroids.** The VLT/SPHERE disk-resolved images of about forty the largest main belt asteroids unveiled strikingly different worlds, which illustrates the complex compositional and geological diversity of this population of objects. In this talk, I will present physical properties of these objects that were observed within the recently completed ESO's large observing campaign. I will report on differences found between shapes of these asteroids, especially with respect to their sizes, multiplicity, and bulk composition and discuss possible implications for their origin.

**Malte Kampschulte: Friction from a dissipation point of view.** In continuum mechanics, friction traditionally occurs in two distinct forms, internally, as a bulk, "dissipation"-type term and externally, in the form of non-trivial boundary conditions. The aim of this talk is to present some thoughts on how both of these effects can be modelled by a single dissipation functional in terms of an energy-dissipation framework and how this approach can be used in proving existence of solutions to dynamic fluid structure-interaction problems involving non-Newtonian fluids and limited-slip boundary conditions.

**Michal Pavelka: Lack-of-fit reduction: From GENERIC to GENERIC.** Consider a GENERIC system, where evolution of the state variables is given by a Poisson bracket, energy, entropy, and a dissipation potential. Having a projection to a set of less detailed state variables. Is the evolution of the less detailed variables also GENERIC? We propose a general reduction method producing GENERIC-type evolution of the less detailed variables while approximating the detailed evolution by minimizing a lack-of-fit Lagrangian. We obtain the less detailed Poisson bracket, energy, entropy, and dissipation potential. A special feature is that we can obtain irreversible evolution even from purely reversible (Hamiltonian) equations.

**Sebastian Schwarzacher: Navier-Stokes-Fourier fluids interacting with elastic shells.** In the lecture we introduce a concept of weak solutions for an interaction of an elastic shell with a compressible heat-conducting fluid. The solid is assumed to be elastic, hence no heat is produced by the motion of the solid. In particular it is shown how energy equality is established, which shows that the system is closed (meaning that the shell is (as assumed) isolating).

**Stefano Pozza: Functions of Rational Krylov Space Matrices and their Decay Properties.** Rational Krylov subspaces have become a fundamental ingredient in numerical linear algebra methods associated with reduction strategies. Nonetheless, many structural properties of the reduced matrices in these subspaces are not fully understood. We advance in this analysis by deriving bounds on the entries of rational Krylov reduced matrices and of their functions that ensure an a-priori decay of their entries as we move away from the main diagonal. As opposed to other decay pattern results in the literature, these properties hold in spite of the lack of any banded structure in the considered matrices.

**Gianmarco Sperone: Unrestricted deformations of thin elastic structures interacting with fluids.** We consider a fluid-structure interaction model in which a two-dimensional cavity, that contains in its middle section a 1D viscoelastic beam, is filled with two fluids in motion that deform the beam separating them, allowing for both normal and tangential displacements of the beam, whereas the remaining walls of the cavity remain fix in time. In collaboration with Sebastian Schwarzacher and Malte Kampschulte (both at the Charles University in Prague), we intend to prove the existence of weak solutions to this coupled problem, by following the variational approach previously introduced by Benešová-Kampschulte-Schwarzacher in 2020.

**Karel Tůma: Light-activated shape memory polymers: stent simulation.** In this talk, we will talk about a new smart material with a shape memory, where the memory is activated with radiation. It is called light-activated shape memory polymer. Up to now in most materials with a memory effect such as shape memory alloys or shape memory polymers, the memory was activated/deactivated with the change of temperature. We present a simple mechanical model to describe the behavior of this material and with this model, we perform three-dimensional simulations of stents using finite element code FEniCS.