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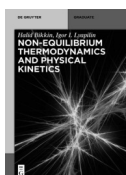
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Multiscale Thermo-Dynamics

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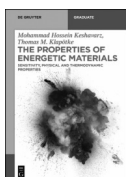


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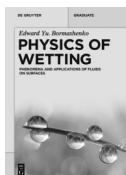


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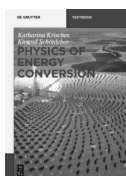


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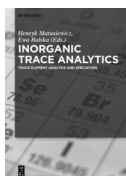


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M.P. and V.K. dedicate this book to their teachers František Maršík and Miroslav Grmela, to their irreplaceable families, to Zuzana, Adam, Markéta, Dana, Jakub, Anna, and Barbora, and to their relatives. M.G. dedicates this book to his students Michal Pavelka and Václav Klika, and to Viera, Sonia, Sylvia and grandchildren.

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Preface

The matter was completely clear. Schweik's spontaneous utterance made it unnecessary to ask a whole lot of questions, and there remained only some of the most important ones, the answers to which were to corroborate Schweik's real opinion, thus:

"Is radium heavier than lead?"

"I've never weighed it, sir," answered Schweik with his sweet smile.

"Do you believe in the end of the world?"

"I have to see the end of the world first," replied Schweik in an offhand manner, *"but I'm sure it won't come my way tomorrow."*

"Could you measure the diameter of the globe?"

"No, that I couldn't, sir," answered Schweik, *"but now I'll ask you a riddle, gentlemen. There's a three-storied house with eight windows on each story. On the roof there are two gables and two chimneys. There are two tenants on each story. And now, gentlemen, I want you to tell me in what year the house porter's grandmother died?"*

The medical authorities looked at each other meaningfully, but nevertheless one of them asked one more question:

"Do you know the maximum depth of the Pacific Ocean?"

"I'm afraid I don't, sir," was the answer, *"but it's pretty sure to be deeper than what the river is just below Prague."*

The chairman of the commission curtly asked, *"Is that enough?"* But one member inquired further:

"How much is 12 897 times 13 863?"

"729," answered Schweik without moving an eyelash.

"I think that's quite enough," said the chairman of the commission. *"You can take this prisoner back to where he came from."*

"Thank you, gentlemen," said Schweik respectfully, *"it's quite enough for me, too."*

– Jaroslav Hašek (translated by Paul Selver), *The Good Soldier Švejk* [1].

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