ESTIMATES FOR THE HOPF INVARIANT IN CRITICAL FRACTIONAL SOBOLEV SPACES

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The Brouwer degree classifies the homotopy classes of mappings from a sphere into itself. Bourgain, Brezis and Mironescu have obtained some linear estimates of the degree of a mapping by any critical first-order or fractional Sobolev energy. Similarly, maps from the three-dimensional sphere to the two-dimensional spheres are classified by their Hopf invariant. Thanks to the Whitehead formula, Rivière has proved a sharp nonlinear control of the Hopf invariant by the first-order critical Sobolev energy. I will explain how a general compactness argument implies that sets that have bounded critical fractional Sobolev energy have bounded Hopf invariant and how we are obtaining in collaboration with Armin Schikorra sharp nonlinear estimates in critical fractional Sobolev spaces with order is close to 1.