(1a) $\lim\limits_{x\to 0} \dfrac{e^{ax}-e^{bx}}{\sin ax - \sin bx}$; $a \neq b \in \mathbb{R}$.

$f(x) = e^{ax} - e^{bx} \to 1-1=0$, $x\to 0$

$g(x) = \sin ax - \sin bx \to 0$, $x \to 0$

Pozn.: $g'(0) = a\cos 0 - b\cos 0 = a-b \neq 0$

$\Rightarrow g(x) \neq g(0) = 0$ na jistém $P(0)$.

---

l'Hosp. asyp. „$\frac{0}{0}$": $\dfrac{f'(x)}{g'(x)} = \dfrac{a\cdot e^{ax} - b e^{bx}}{a\cdot\cos ax - b\cos bx} \longrightarrow \dfrac{a-b}{a-b} = 1.$

(1b) $\lim\limits_{x\to\infty} \dfrac{\ln(x^2-x+1)}{\ln(x^{10}+x-10)}$;

$x^{10}+x-10 = x^{10}\left(1+\frac{1}{x^9}-\frac{10}{x^{10}}\right) \to \infty^{10}\left(1+\frac{1}{\infty^9}-\frac{10}{\infty^{10}}\right) = \infty$

$\ln y \to \infty$, $y\to\infty$: VoLSF: čitatel $\to +\infty$

---

l'Hosp. asym. $\dfrac{\text{cosi}}{\infty}$:

$$\frac{f'(x)}{g'(x)} = \frac{\dfrac{2x-1}{x^2-x+1}}{\dfrac{10x^9+1}{x^{10}+x-10}} = \frac{(2x-1)(x^{10}+x-10)}{(10x^9+1)(x^2-x+1)} \cdot \frac{\frac{1}{x^{11}}}{\frac{1}{x^{11}}}$$

$$= \frac{\left(2-\frac{1}{x}\right)\left(1+\frac{1}{x^{10}}-\frac{10}{x^9}\right)}{\left(10+\frac{1}{x^9}\right)\left(1-\frac{1}{x}+\frac{1}{x^2}\right)} \longrightarrow \frac{2\cdot 1}{10\cdot 1} = \frac{1}{5}, \ x\to+\infty$$

dle VoAL

(1c) $\lim\limits_{x\to 0} \dfrac{\operatorname{arctg} x - x}{x - \sin x}$; zřejmě l'Hosp. typ „$\frac{0}{0}$"

$$\frac{f'(x)}{g'(x)} = \frac{\frac{1}{1+x^2} - 1}{1 - \cos x} = \underbrace{\frac{1}{1+x^2}}_{\to 1} \cdot \underbrace{\frac{-x^2}{1-\cos x}}_{\to -2} \to -2$$

dle VOAL a základní limity: $\frac{1-\cos x}{x^2} \to \frac{1}{2}$, $x \to 0$

(1d) $\lim\limits_{x\to +\infty} x\left(\operatorname{arctg} x - \frac{\pi}{2}\right) = \lim\limits_{x\to+\infty} \dfrac{\operatorname{arctg} x - \frac{\pi}{2}}{\frac{1}{x}}$

l'Hosp. „$\frac{0}{0}$": $\dfrac{f'(x)}{g'(x)} = \dfrac{\frac{1}{1+x^2}}{-\frac{1}{x^2}} = -\dfrac{x^2}{1+x^2} = -\dfrac{1}{\frac{1}{x^2}+1} \to -1$

$x \to +\infty$.

(1e) $\left(\frac{\sin x}{x}\right)^{\frac{1}{x^2}} = \exp h(x)$; $h(x) = \dfrac{\ln\left(\frac{\sin x}{x}\right)}{x^2} = \dfrac{\ln \sin x - \ln x}{x^2}$

$\frac{\sin x}{x} \to 1$; $f(x) = \ln\left(\frac{\sin x}{x}\right) \to \ln 1 = 0$ $\boxed{x \to 0}$

$g(x) = x^2 \to 0$;

l'Hosp. typ „$\frac{0}{0}$": $\dfrac{f'(x)}{g'(x)} = \dfrac{\frac{\cos x}{\sin x} - \frac{1}{x}}{2x} = \dfrac{x\cos x - \sin x}{2x^2 \sin x}$

... opět „$\frac{0}{0}$": $\dfrac{f''(x)}{g''(x)} = \dfrac{\cos x - x\sin x - \cos x}{4x\sin x + 2x^2 \cos x}$

$$= -\frac{x\cdot \sin x}{4x\sin x + 2x^2\cos x} = \frac{-1}{4 + 2\cos x\left(\frac{x}{\sin x}\right)} \to \frac{-1}{4+2\cdot 1\cdot 1} = -\frac{1}{6}$$

$x \cdot \sin x \neq 0$ na $P(0)$

(18) $\left(\frac{4\,\mathrm{arctg}\,x}{\pi}\right)^{\frac{1}{\arccos^2 x}} = \exp h(t)$; $\quad h(t) = \dfrac{\ln\left(\frac{4\,\mathrm{arctg}\,x}{\pi}\right)}{\arccos^2 x}$

$x \to 1-$

$\arccos x \to \arccos 1 = 0, \; x \to 1-$

$\mathrm{arctg}\,x \to \mathrm{arctg}\,1 = \frac{\pi}{4}$, tj. typ „$\frac{0}{0}$"

$$f'(t) = \left(\ln(\mathrm{arctg}\,x) + \ln\frac{4}{\pi}\right)' = \frac{1}{(1+x^2)} \cdot \frac{1}{\mathrm{arctg}\,x}$$

$$g'(t) = \left(\arccos^2 x\right)' = 2\arccos x \cdot \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{f'(t)}{g'(t)} = \underbrace{\frac{1}{1+x^2}}_{\to \frac{1}{2}} \cdot \underbrace{\frac{1}{\mathrm{arctg}\,x}}_{\to \frac{4}{\pi}} \cdot \left(\frac{-2\arccos x}{\sqrt{1-x^2}}\right)^{-1};$$

zbývá: $\lim\limits_{x\to 1-} \dfrac{\arccos x}{\sqrt{1-x^2}}$, opět typ „$\frac{0}{0}$":

$$\frac{\tilde{f}'(t)}{\tilde{g}'(t)} = \frac{(\arccos x)'}{\left(\sqrt{1-x^2}\right)'} = \frac{-\frac{1}{\sqrt{1-x^2}}}{\frac{-x}{\sqrt{1-x^2}}} = \frac{1}{x} \to 1; \quad x \to 1-.$$

---

celkem: $h(t) \to \frac{1}{2} \cdot \frac{4}{\pi} \cdot (-2)^{-1} = \underline{\underline{-\frac{1}{\pi}}}.$

(1g) $\frac{x^a}{e^{bx}}$; $a, b > 0$ zadané ; $x \to +\infty$

$e^y \to +\infty$, $y \to +\infty$; tedy typ $\frac{\text{něco}}{+\infty}$.

l'Hosp.: $\frac{a x^{a-1}}{b \cdot e^{bx}}$; – stejný typ, opakuji n krát,

kde $n > a$.

$$\underbrace{\frac{a \cdot (a-1) \cdots (a-n+1)}{b^n}}_{c} \frac{x^{a-n}}{e^{bx}} \to c \cdot \frac{0}{+\infty} = 0$$

neboť $x^{a-n} \to 0$ (záporný exponent mocniny)
$x \to +\infty$

(1h) $(\sin x)^{\frac{\pi}{\ln x}} = \exp h(x)$; $h(x) = \frac{\pi \cdot \ln(\sin x)}{\ln x}$;

$x \to 0+$.

typ $\frac{\text{cosi}}{-\infty}$; l'Hosp.

$$\frac{\pi \cdot \frac{\cos x}{\sin x}}{\frac{1}{x}} = \pi \cdot \cos x \cdot \frac{x}{\sin x} \to \pi \cdot 1 \cdot 1 = \pi$$

dle zákl. limity pro $\sin x$

a spojitosti $\cos x$ v 0.