A1 $= \int_0^{\infty} \left[\frac{e^{-yx}}{x} \right]_{y=b}^{y=a} dx = \int_0^{\infty} \left(\int_b^a - e^{-yx} dy \right) dx = \iint_{\Pi} \left(-e^{-yx} \right) dx dy$

$= -\int_b^a \left(\int_0^{\infty} e^{-yx} dx \right) dy = -\int_b^a \frac{1}{y} dy = \ln\left(\frac{b}{a}\right). \quad a, b > 0$

A2 $= \int_0^{\infty} \left[\frac{\operatorname{arctg}(yx)}{x} \right]_{y=a}^{y=b} dx = \int_0^{\infty} \left(\int_a^b \frac{1}{1+y^2x^2} dy \right) dx$

$= \int_a^b \left(\int_0^{\infty} \frac{dx}{1+y^2x^2} \right) dy = \int_a^b \frac{\pi}{2y} dy = \frac{\pi}{2} \ln\left(\frac{b}{a}\right). \quad ; a, b > 0$

A3 $= \int_0^{\infty} \frac{e^{-x} - e^{-(a+1)x}}{x} dx$ -- viz A1

A4 $= \int_0^{\pi} \left[\frac{\ln(1+y\cos x)}{\cos x} \right]_{y=0}^{y=a} dx = \int_0^{\pi} \left(\int_0^a \frac{dy}{1+y\cos x} \right) dx$

$= \int_0^a \left(\int_0^{\pi} \frac{dx}{1+y\cos x} \right) dy = \int_0^a \frac{\pi}{2} \frac{dy}{\sqrt{1-y^2}} = \frac{\pi}{2} \arcsin a$

$|a| < 1$

$\int_0^{\pi} \frac{dx}{1+y\cos x} = \left| \begin{array}{l} t = \operatorname{tg}\frac{x}{2} \in (0, +\infty) \\ \cos x = \frac{1-t^2}{1+t^2} \\ dx = \frac{2dt}{1+t^2} \end{array} \right| = \int_0^{\infty} \frac{dt}{1+y\frac{1-t^2}{1+t^2}} \cdot \frac{2}{1+t^2}$

$= \int_0^{\infty} \frac{2 \cdot dt}{(1+y)+(1-y)t^2} = \frac{\pi}{2} \cdot \frac{1}{\sqrt{1-y^2}} ;$

dle vzorce: $\int_0^{\infty} \frac{dt}{A+Bt^2} = \frac{\pi}{2\sqrt{AB}}$

$A, B > 0.$

A5

$$\int_0^1 \frac{x^a - x^b}{\ln x}\,dx = \int_0^1 \left[\frac{x^y}{\ln x}\right]_{y=b}^{y=a} dx = \int_0^1 \left(\int_b^a x^y\,dy\right) dx$$

$$= \int_0^1 \left(\int_b^a x^y\,dx\right) dy = \int_b^a \frac{1}{y+1}\,dy = \ln\frac{a+1}{b+1}; \quad a, b > -1$$

A6

$$= \int_0^\infty \left[\frac{\operatorname{arctg} yx}{x(1+x^2)}\right]_{y=0}^{y=a} = \int_0^\infty \left(\int_0^a \frac{dy}{(1+y^2x^2)(1+x^2)}\right) dx$$

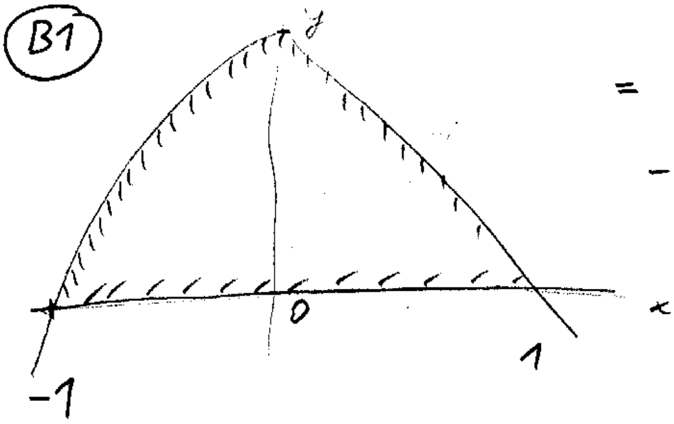
$$= \int_0^a \left(\int_0^\infty \frac{dx}{(1+x^2)(1+y^2x^2)}\right) dy = \int_0^a \frac{1}{1-y^2} \cdot \frac{\pi}{2}(y-1)\,dy$$

$$= \int_0^a \frac{\pi}{2}\,\frac{1}{1+y}\,dy = \frac{\pi}{2}\ln(1+a); \quad a > 0.$$

$$\frac{1}{(1+x^2)(1+y^2x^2)} = \frac{1}{y^2-1}\left\{\frac{y^2}{1+x^2y^2} - \frac{1}{1+x^2}\right\}$$

$$\frac{1}{y(1+y)} = \frac{1}{y} - \frac{1}{1+y} = \ln\left(\frac{y}{1+y}\right)$$

B1



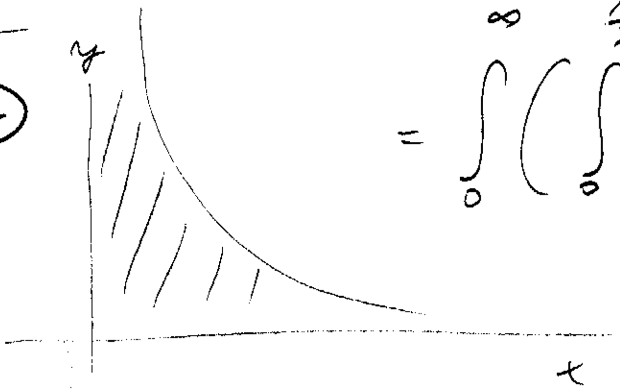
$$= \int_{-1}^{0}\left(\int_{0}^{1-x^2} x^2+y \, dy\right)dx + \int_{0}^{1}\left(\int_{0}^{1-x} x^2+y\, dy\right)dx$$

$$= \int_{-1}^{0} \frac{1-x^4}{2}\,dx + \int_{0}^{1} \frac{-2x^3+3x^2-2x+1}{2}\,dx$$

ověření: omezený integrand i oblast

$$= \frac{2}{5} + \frac{1}{4}.$$

B2

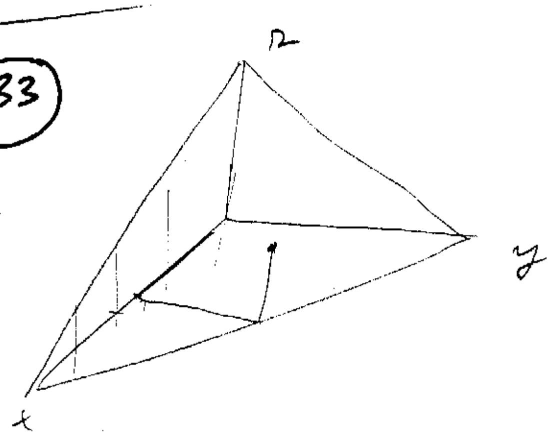


$$= \int_{0}^{\infty}\left(\int_{0}^{\frac{1}{x}} e^{-xy}\, dy\right)dx = \int_{0}^{\infty} \frac{1}{x}\cdot(1-e^{-1})\,dx$$

$$= +\infty.$$

ověření: integrand ≥ 0

B3



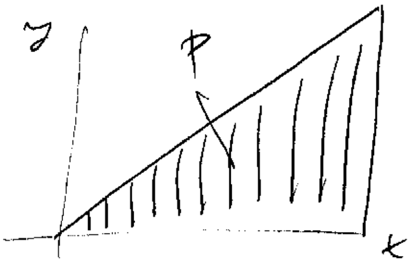
$$= \int_{0}^{1}\left(\int_{0}^{1-x}\left(\int_{0}^{1-x-y} \frac{dz}{(1+x+y+z)^3}\right)dy\right)dx$$

$$= \int_{0}^{1}\left(\int_{0}^{1-x} \frac{1}{2}\left(\frac{1}{(x+y+1)^2} - \frac{1}{8}\right)dy\right)dx$$

$$= \int_{0}^{1} \frac{x^2-4x+3}{16x+16}\,dx = \frac{16\ln 2 - 9}{32}.$$

B4 $\quad M = \{0 \le z \le xy;\ 0 \le y \le x \le 1\}$

„podstava": $x \in (0,1)$
$y \in (0,x)$



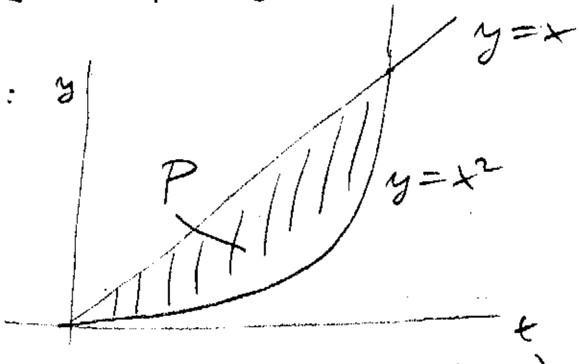
$$= \iint_P \left(\int_0^{xy} xy^2z^2 \right) dx\,dy$$

$$= \int_0^1 \left(\int_0^x \left(\int_0^{xy} \left(xy^2z^2 \right) dz \right) dy \right) dx$$

$$= \int_0^1 \left(\int_0^x \tfrac{1}{3} x^4 y^5 \, dy \right) dx = \int_0^1 \frac{x^{10}}{18} dx = \frac{1}{11 \cdot 18}.$$

C1 $\quad x^2+y^2 \le z \le 2(x^2+y^2)$

„podstava":

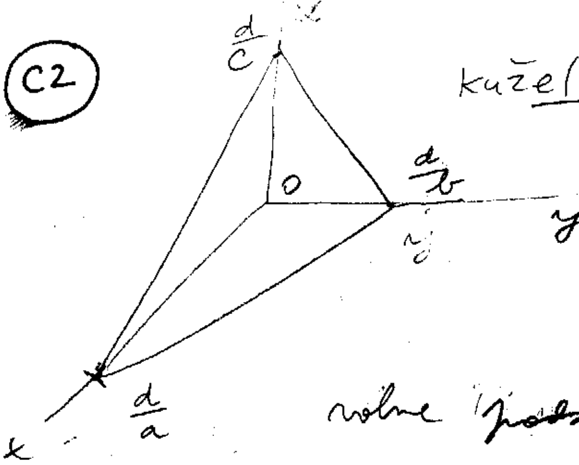


$$\lambda_3(M) = \iiint_M 1 \, dx\,dy\,dz = \iint_P \left(\int_{x^2+y^2}^{2(x^2+y^2)} dz \right) dx\,dy$$

$$= \iint_P (x^2+y^2) \, dx\,dy = \int_0^1 \left(\int_{x^2}^{x} (x^2+y^2) \, dy \right) dx$$

$$= \int_0^1 \tfrac{1}{3} \left(4x^3 - x^6 - 3x^4 \right) dx = \frac{3}{35}.$$

(C2)



kužel: $V = \frac{1}{3}\lambda_2(P)\cdot v$

P -- podstava

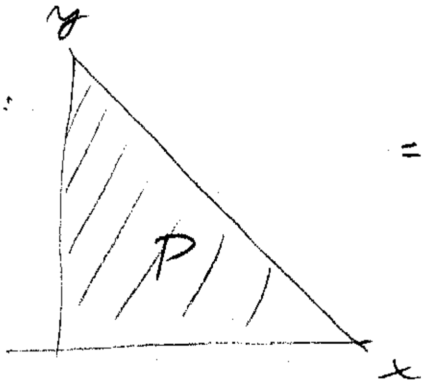
v -- (kolmá) výška.

volme podstavu v rovině xy:

$$\left.\begin{array}{l}\lambda_2(P) = \frac{1}{2}\cdot\frac{d}{a}\cdot\frac{d}{b}, \\ v = \frac{d}{c}\end{array}\right\} V = \frac{1}{6}\frac{d^3}{abc}$$

(C3)

podstava:



$$= \iint_P \left(\int_{xy}^{x+y} 1\,dz \right) dx\,dy$$

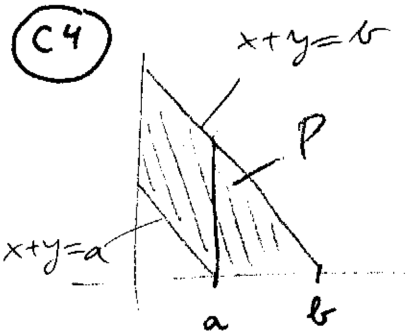
platí pro $x, y \in (0,1)$

$xy \le x \le x+y$.

$$= \iint_P (x+y-xy)\,dx\,dy = \int_0^1 \left(\int_0^{1-x} x+y-xy\,dy \right) dx$$

$$= -\frac{1}{2}\int_0^1 x^3 - x^2 + x - 1\,dx = \frac{7}{24}$$

(C4)



$$= \iint_P \left(\int_{-\sqrt{xy}}^{\sqrt{xy}} dz \right) dx\,dy = 2\iint_P \sqrt{xy}\,dx\,dy$$

$$x \quad = \int_0^a \left(\int_{a-x}^{b-x} \sqrt{xy}\,dy \right) dx + \int_a^b \left(\int_0^{b-x} \sqrt{xy}\,dy \right) dx$$

$$= \int_0^a \frac{2}{3} x^{\frac{1}{2}} \left\{ (b-x)^{3/2} - (a-x)^{3/2} \right\} dx + \int_a^b \frac{2}{3} x^{\frac{1}{2}} (b-x)^{3/2}\,dx = \;??$$