cvičení 21.4.2020

(https://cesnet.zoom.us/j/8305603965
Meeting ID: 830 560 3965)

Plán:
1) d.ú. série 2 (vzorové řešení)
2) dotazy, připomínky (na volné téma)
3) modelování COVID-19 (aplikace SIR, atd.)

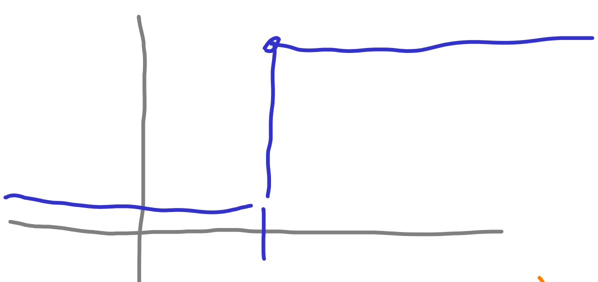
---

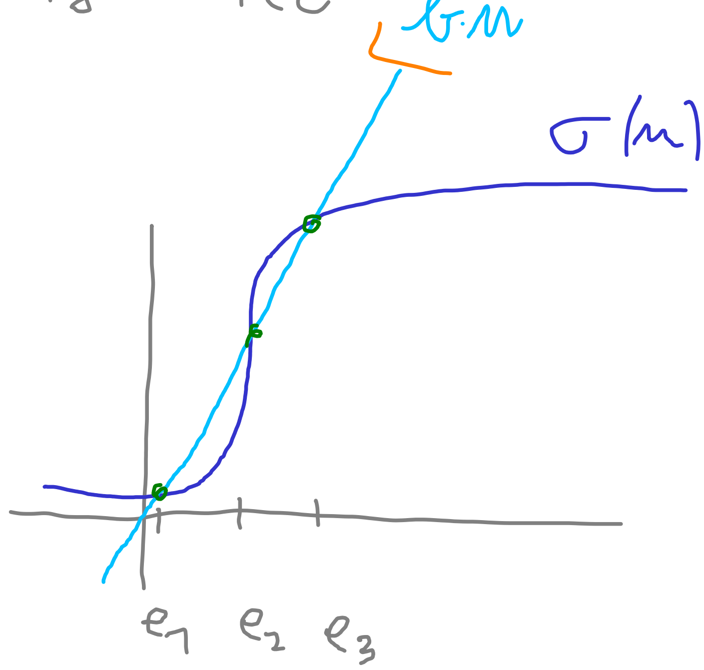
ad 1)

$$\partial_{tt}u + a\,\partial_t u + bu = \boxed{\sigma(u)} + k\,\partial_{xx}u$$

$$u = u(t,x)$$

$$\sigma(u) = \frac{\alpha}{\alpha + e^{-\beta u}}$$ „sigma fce"





tvrdím: $\exists$ 3 ekv. *)

$$0 < e_1 < e_2 < e_3$$

Dk: nepřímo:

*) pro vhodná b, α, β

lze spočst: $\begin{cases} \sigma'(u) > 0 \\ \sigma(u) \to H(u) = \begin{cases} 0, & x \leq 0 \\ 1, & x > 0 \end{cases} \\ (\beta \to +\infty,\ \alpha \to 0+) \\ \exists!\, m_0 > 0 : \text{maximum } \sigma'(u) \\ \quad (\text{inflexní bod}) \end{cases}$

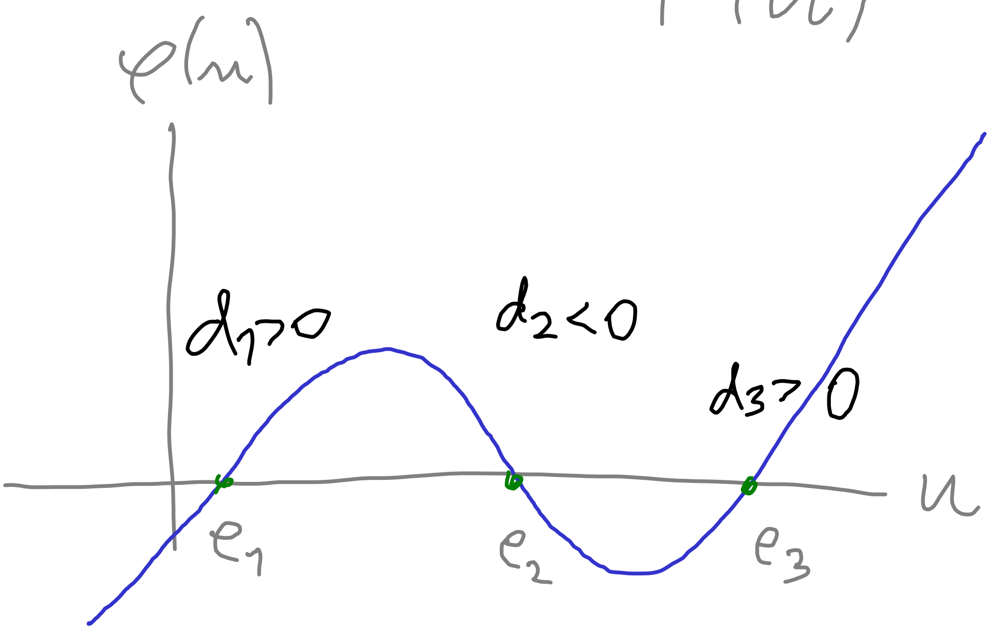
ad D.ú.2: $u = u(t,x) = U(x - \underline{\underline{c}}\,t)$

U — profil, c — rychlost vlny

dosaď:

$$c^2 U'' - acU' + bU = \sigma(U) + \varepsilon U''$$

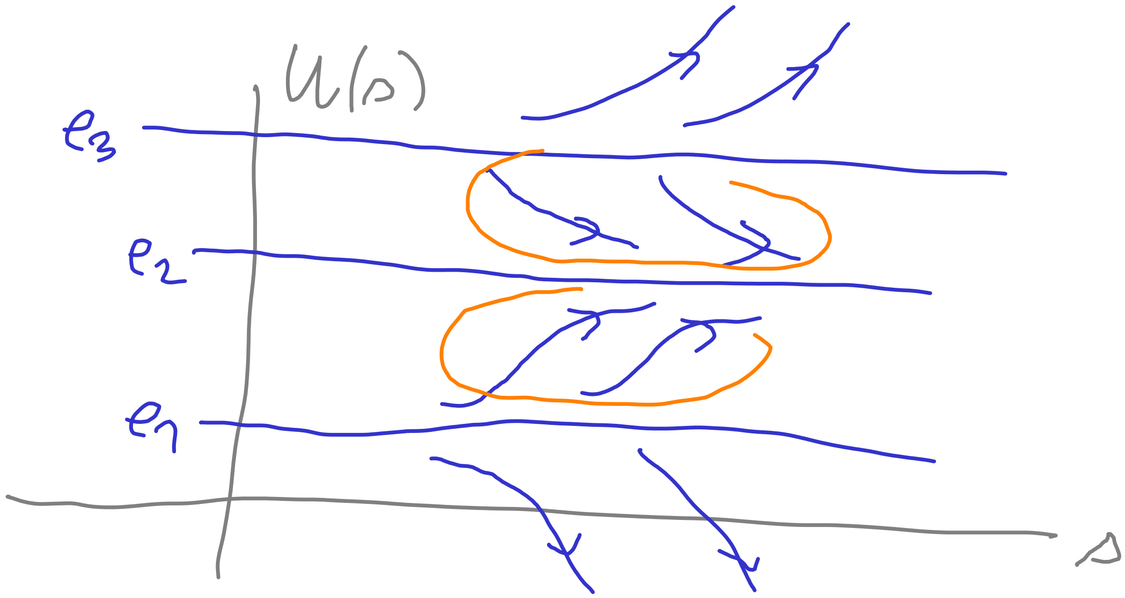
$$\boxed{(c^2 - \varepsilon)U'' - acU' + \underbrace{bU - \sigma(U)}_{\varphi(U)} = 0}$$



označ: $d_i = \varphi'(e_i)$

(i) $c^2 - q = 0 \Rightarrow$ $\boxed{u' = \varphi(u)}$ $u = u(s)$

BdNO $ac = 1$



(ii) $a = 0$:

(BdNO: $c^2 - q = 1$)

$$u'' + \varphi(u) = 0 \quad / \, u'$$

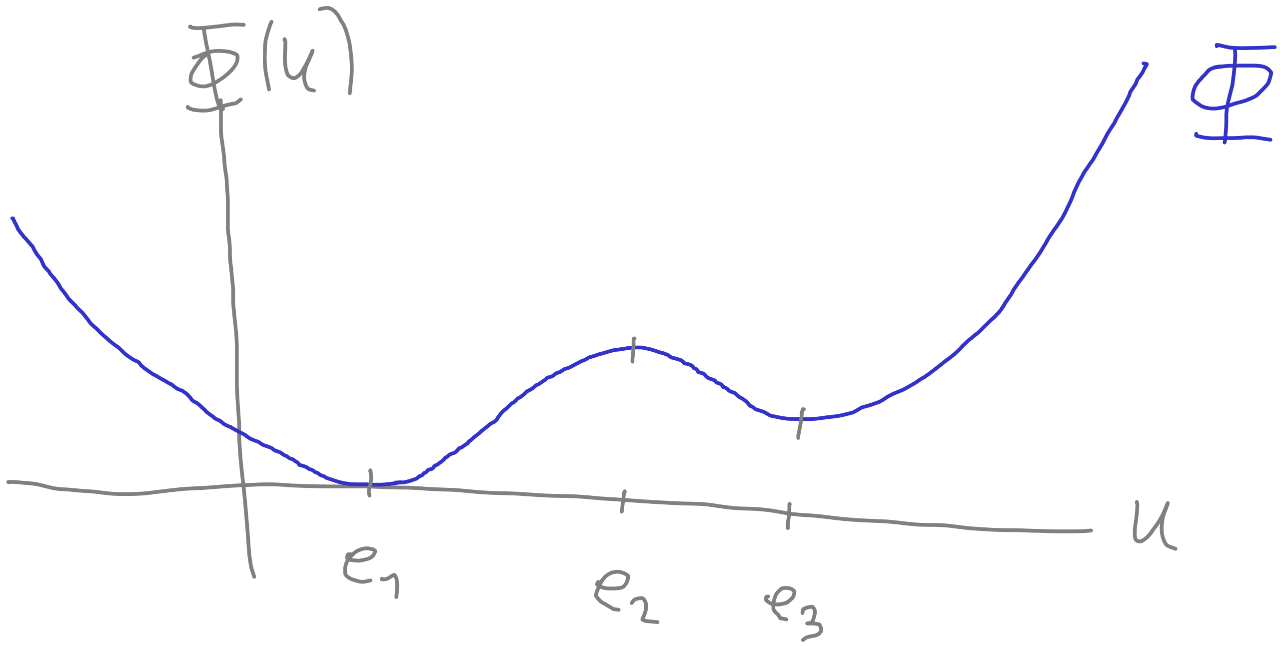
$$\begin{pmatrix} u' = v \\ v' = -\varphi(u) \end{pmatrix}$$

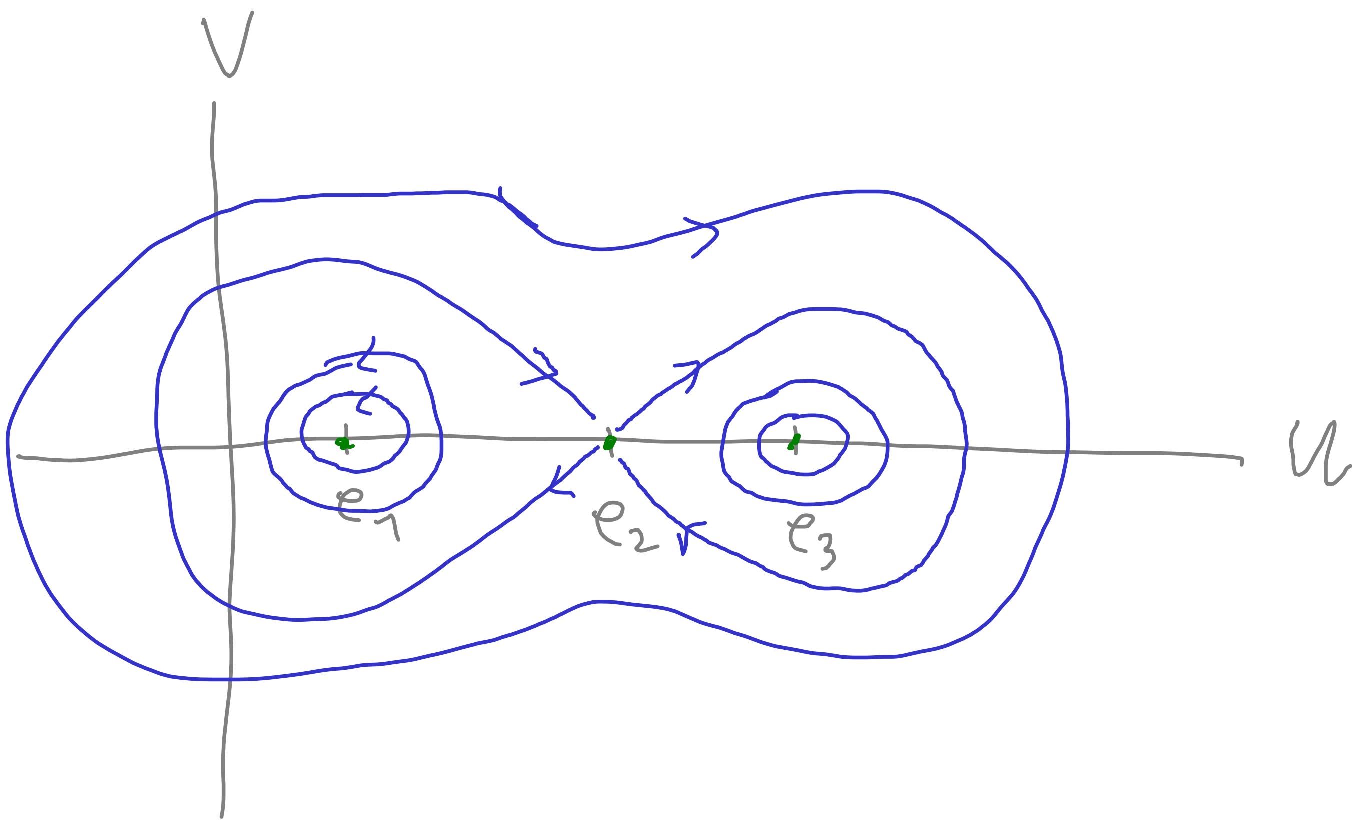
$$\frac{1}{2} v^2 + \Phi(u) = c$$

(1. integrál)

$\Phi = \int \varphi$

$$\boxed{u'' = -\Phi'(u)}$$



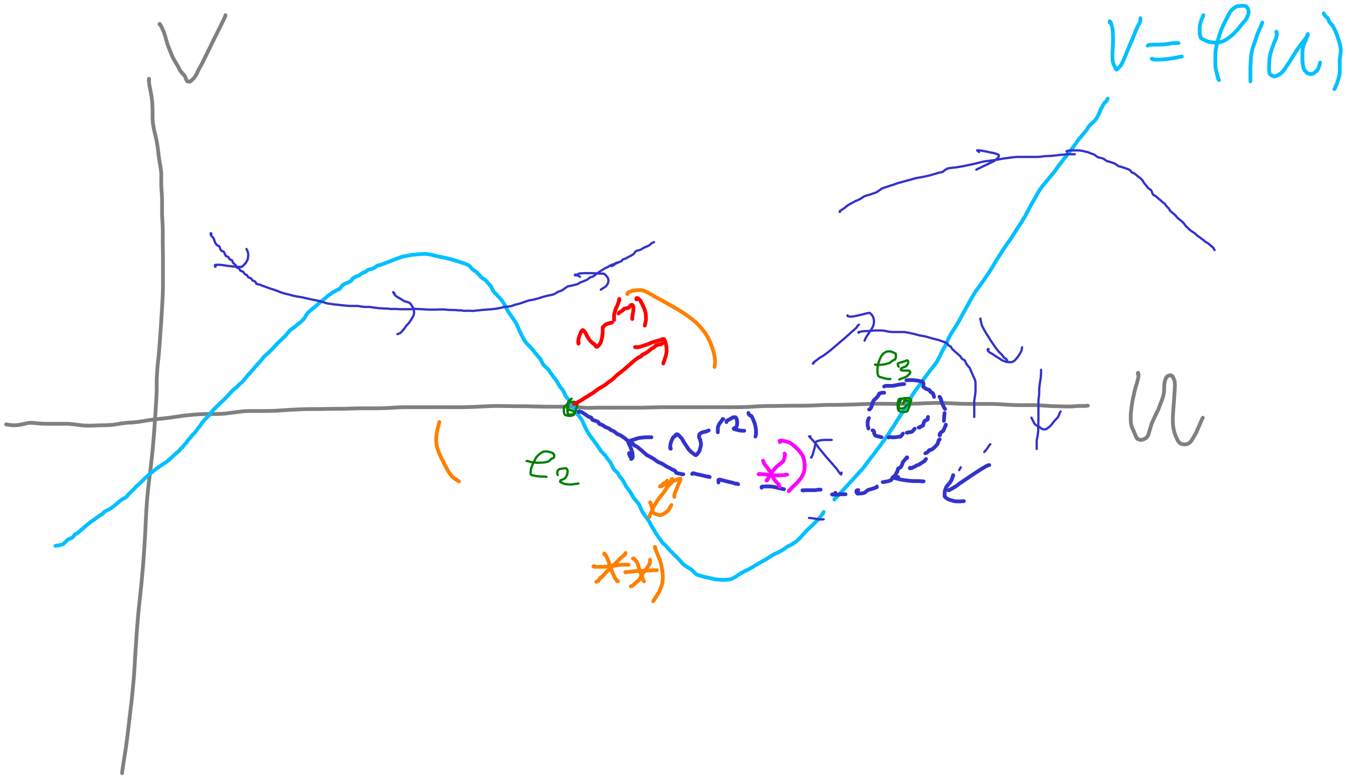


(iii) obecný případ: $u'' - u' + \varphi(u) = 0$

(BÚNO: $c^2 - \varkappa_2 = 1$
$ac = 1$)

$$\begin{cases} u' = v \\ v' = v - \varphi(u) \end{cases}$$

$$\begin{pmatrix}u\\v\end{pmatrix}' = \mathcal{F}\begin{pmatrix}u\\v\end{pmatrix}$$



pomocný výpočet: linearizace v bodech e_2, e_3:

$$DF = \begin{pmatrix} 0, & 1 \\ -\varphi'(u), & 1 \end{pmatrix} \qquad \text{tj. } A_2 = \begin{pmatrix} 0, & 1 \\ -d_2, & 1 \end{pmatrix} \qquad d_2 < 0$$

$$\chi(\lambda) = \det(\lambda I - A_2) = \begin{vmatrix} \lambda, & -1 \\ d_2, & \lambda - 1 \end{vmatrix} = \lambda^2 - \lambda + d_2 \quad {*)}$$

$$\lambda_{1,2} = \frac{1 \pm \sqrt{D}}{2}, \quad D = 1 - 4d_2 > 1$$

$$\Rightarrow \quad \lambda_1 > 0 > \lambda_2$$

další výpočet: „sedlový bod"

$$\lambda_i \ \cdots \ v^{(i)} = \begin{pmatrix} 1 \\ \lambda_i \end{pmatrix}$$

ověření: $\begin{pmatrix} 0, & 1 \\ -d_2, & 1 \end{pmatrix} \begin{pmatrix} 1 \\ \lambda_i \end{pmatrix} = \begin{pmatrix} \lambda_i \\ -d_2 + \lambda_i \end{pmatrix}$

ad **) $\overset{??}{\lambda_2 > d_2} \iff \dfrac{1-\sqrt{1-4d_2}}{2} > d_2$ $\quad$ " λ_i^2

$$\sqrt{1-4d_2} < 1-2d_2$$

$$\sqrt{1+2\alpha} < 1+\alpha \quad (\alpha = -2d_2 > 0)$$

$$1+2\alpha < (1+\alpha)^2 = 1+2\alpha+\alpha^2$$

$$0 < \alpha^2 \quad \text{o.k.}$$

ad e_3)

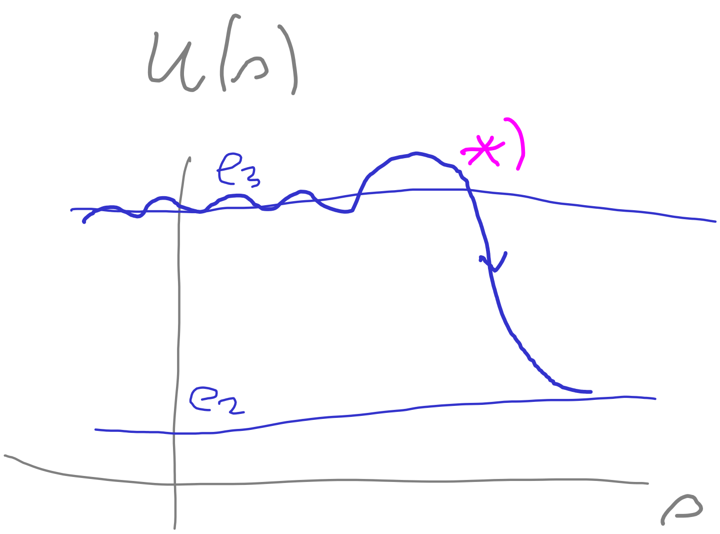
$$A_3 = \begin{pmatrix} 0, & 1 \\ -d_3, & 1 \end{pmatrix} \Rightarrow \lambda_{1,2} = \frac{1 \pm \sqrt{D}}{2}$$

$$D = 1 - 4d_3 < 0$$

$d_3 > 0$, pokud $\overset{??}{:}$ $d_3 > \frac{1}{4}$

$$\Rightarrow \lambda_{1,2} = \frac{1}{2} \pm i\omega$$

(nestabilní spirála)



ad 3) sir-model (SIR – bezrozměrná verze)

$$S' = -\beta S I \qquad \alpha^{-1} \text{ ... doba nemoci}$$

$$I' = \beta S I - \alpha I \qquad \beta \text{ .... míra soc. kont.}$$

$$R' = \alpha I$$

$$N' = 0 \quad (\text{celá populace})$$

$$\left.\begin{aligned} s &= \frac{S}{N} \\ i &= \frac{I}{N} \\ r &= \frac{R}{N} \end{aligned}\right\} \in (0,1)$$

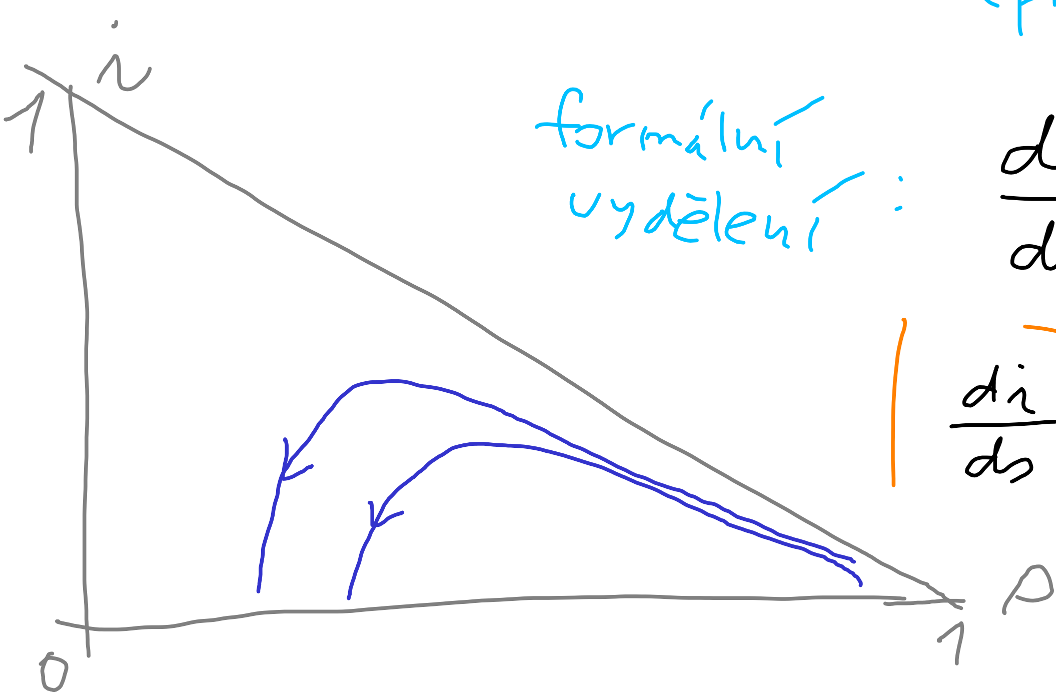
$$\tau = \alpha \cdot t = \frac{t}{\alpha^{-1}}$$

$$\boxed{\begin{aligned} s' &= -\rho_0 s i \\ i' &= \rho_0 s i - i \\ r' &= i \end{aligned}}$$

$$\rho_0 = \frac{\beta}{\alpha} = R_0$$

základní repr. číslo

$$s + i + r \equiv 1$$



formální vydělení:

$$\frac{di}{ds} = \frac{i(\rho_0 s - 1)}{-\rho_0 s i}$$

$$\boxed{\frac{di}{ds} = -1 + \frac{1}{\rho_0 s}}$$

$$\Rightarrow \quad \underbrace{i_\infty - i_0}_{=0} + s_\infty - \underbrace{s_0}_{=1} = \frac{1}{\rho_0} \cdot \ln\left(\frac{s_\infty}{\underbrace{s_0}_{=1}}\right)$$

$$\rho_0 = \frac{-\ln s_\infty}{1 - s_\infty}$$

$$\boxed{\rho_0 = -\frac{\ln(1 - r_\infty)}{r_\infty}}$$

kde $r_\infty = 1 - s_\infty$

„míra dopadu"
(attack rate)

tabulka:

r_∞	ρ_0	
10%	1.05	nCoV (+karanténa)
20%	1.12	
⋮		
50%	1.39	← chřipka (bez karantény)
70%	1.72	