Příklad: vězňovo dilemma

$$A = \begin{pmatrix} 2, & -1 \\ 4, & 0 \end{pmatrix} \xrightarrow[\text{lizace}]{\text{norma}} \begin{pmatrix} 0 & -1 \\ 2 & \boxed{0} \end{pmatrix}$$ ← jediné N.e.

Rep. dyn.?

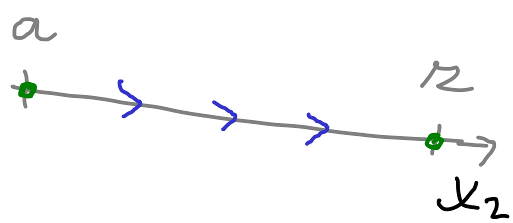
$$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$ x_1 – altr., x_2 – zrádné

$$Ax = \begin{pmatrix} -x_2 \\ 2x_1 \end{pmatrix} = \begin{pmatrix} \pi_1(x) \\ \pi_2(x) \end{pmatrix}$$

$$\pi = x \cdot Ax = x_1 x_2$$

$$\Rightarrow \quad x_1' = x_1(-x_2 - x_1 x_2) = x_1 x_2 (-1 - x_1)$$

$$x_2' = x_2(2x_1 - x_1 x_2) = x_2 x_1 (2 - x_2)$$



$x_1' < 0$
$x_2' > 0$ uvnitř Δ

($\nexists$ N.e.)

$$\Delta = \{(x_1, x_2);\ x_1 \geq 0,\ x_2 \geq 0,\ x_1 + x_2 = 1\}$$

... altruismus ?!

varianta: opakování hry $\quad p \in (0,1)$

délka hry: $L = \dfrac{1}{1-p}$

strategie:
- a (vždy A)
- r (-"- Z)
- σ (oplácející/odpouštějící) "Tit for tat"

$$\pi(a,a) = \underbrace{2 + p(2 + p(2 + \dots))}_{2\cdot(1+p+\cancel{p^2}+\dots)} \cdot \underbrace{\frac{1}{L}}_{\cancel{(1-p)}} = 2 = \pi(A,A)$$

výplatní matice

$$\begin{array}{c|ccc} & a & r & \sigma \\ a & 2 & -1 & 2 \\ r & 4 & 0 & 4(1-p) \\ \sigma & 2 & (p-1) & 2 \end{array}$$

$$\pi(\sigma, r) = -1 + p(0 + \dots) / \frac{1}{1-p}$$
$$= p-1$$
$$\pi(r,\sigma) = (4+0)(1-p) = 4(1-p)$$

BÚNO: $p = \frac{3}{4}$

$$\begin{pmatrix} 2 & -1 & 2 \\ 4 & 0 & 1 \\ 2 & -\frac{1}{4} & 2 \end{pmatrix} \xrightarrow[\text{lizace}]{\text{norma}} \boxed{\begin{pmatrix} 0 & -1 & 0 \\ 2 & 0 & -1 \\ 0 & -\frac{1}{4} & 0 \end{pmatrix}}$$

Repl. dyn.: $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \begin{array}{l} - a \\ - r \\ - \sigma \end{array}$

$$Ax = \begin{pmatrix} -x_2 \\ 2x_1 - x_3 \\ -\frac{x_2}{4} \end{pmatrix} = \begin{pmatrix} \pi_1(x) \\ \pi_2(x) \\ \pi_3(x) \end{pmatrix}$$

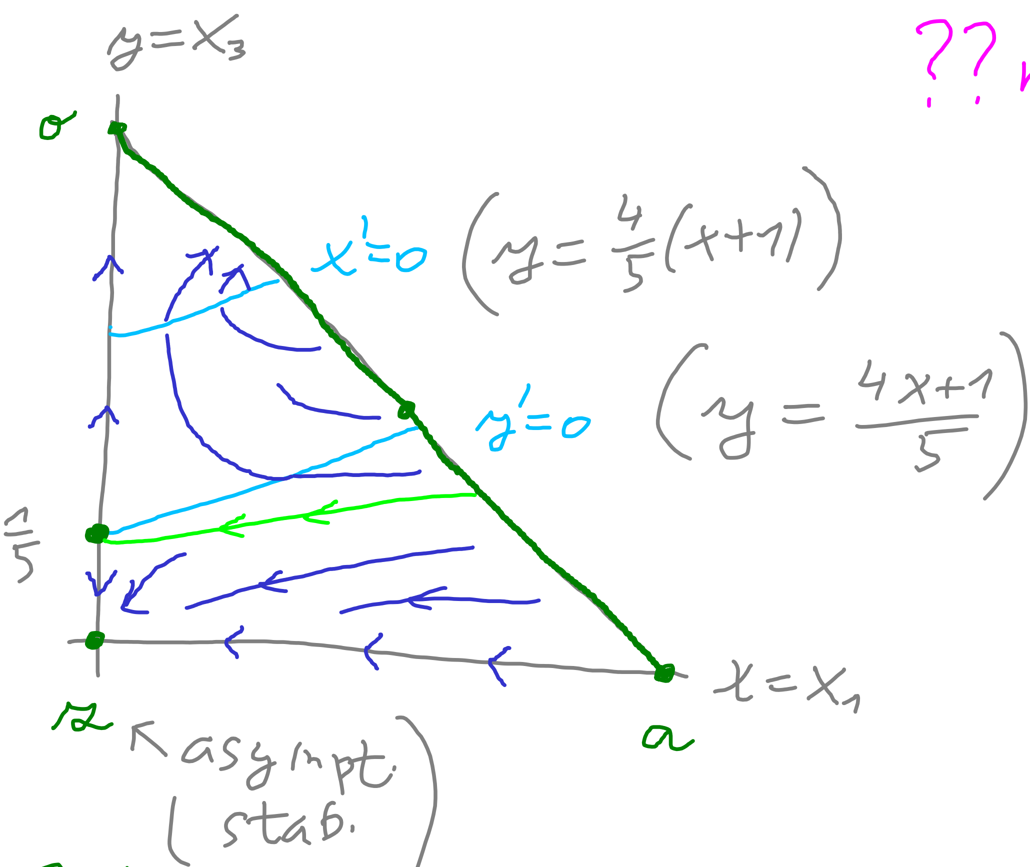
$$\pi(x) = x \cdot Ax = x_2\left(x_1 - \frac{5}{4}x_3\right)$$

$$\boxed{\begin{aligned} x_1' &= \frac{x_1 x_2}{4}(5x_3 - 4x_1 - 4) \\ x_2' &= \frac{x_2}{2}(5x_2x_3 - 4x_3 - 4x_1x_2 + 8x_1) \\ x_3' &= \frac{x_2 x_3}{4}(5x_3 - 4x_1 - 1) \end{aligned}}$$

víme: $x_1+x_2+x_3=1 \Rightarrow$ $x = x_1$
$y = x_3$
$(x_2 = 1-x-y)$

$$\Rightarrow \boxed{\begin{aligned} x' &= \tfrac{1}{4}x(1-x-y)(\underline{5y-4x-4}) \\ y' &= \tfrac{1}{4}y(1-x-y)(\underline{5y-4x-1}) \end{aligned}}$$

??nerobustní dynamika



? stac body

(∄ uvnitř: $\Leftarrow (\operatorname{adj} A)u = \begin{pmatrix} -\frac{1}{4} & 0 & 1 \\ 0 & 0 & 0 \\ -\frac{1}{2} & 0 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3/4 \\ 0 \\ 3/2 \end{pmatrix}$)

Věta 11

varianta: mutace $\quad M = (\mu_{ij})$

μ_{ij} ... pravděpodobnost mutace $j \to i$ (za 1 časů)

$n_i = n_i(t)$ ... počet i-tých čistých strat.

$$x_i(t) = \frac{n_i(t)}{\sum_j n_j(t)}$$

a) diskrétní čas: (mutace)

i-tá složka: $\sum_j \mu_{ij} n_j - \mu_{ji} n_i$ (bilance)

z $\boxed{j}$ $\quad$ do j)

$$n_i \left(1 - \sum_j \mu_{ji}\right)$$ (nemění se)

CELKEM:

koef. růstu

$$n_i(t+1) = n_i(t) + \underbrace{r_i n_i(t) \cdot \left(1 - \sum_j \mu_{ji}\right)}_{\text{replikace}}$$
$$+ \sum_j \left(\mu_{ij} n_j(t) - \mu_{ji} n_i(t)\right)$$

mutace

b) spojitý čas:

$$n_i(t+\tau) = n_i(t) + \tau\left[(\text{repl.}) + (\text{mut.})\right]$$

$$\Longrightarrow$$

$$\boxed{n_i' = r_i n_i \left(1 - \sum_j \mu_{ji}\right) + \sum_j \left(\mu_{ij} n_j - \mu_{ji} n_i\right)}$$

$$x_i' = \left(\frac{n_i}{\sum_j n_j}\right)' = \frac{n_i'}{\sum n_j} - \frac{n_i}{(\sum n_j)^2}\cdot \sum n_j'$$

$$= r_i x_i \left(1 - \sum_j \mu_{ji}\right) + \sum_j \left(\mu_{ij} x_j - \mu_{ji} x_i\right)$$

$$- x_i \left[\sum_j r_j x_j \left(1 - \sum_k \mu_{ki}\right) + 0\right]$$

neboť: celková bilance mutací $=0$

Závěr: obecný model replikace-mutace

$$x_i' = x_i \left(r_i - \sum_j x_j r_j\right)\left(1 - \sum_j \mu_{ji}\right) + \sum_j \left(\mu_{ij} x_j - \mu_{ji} x_i\right)$$

Speciálně:
$$r_i = \pi_i(x)$$
$$u_{ij} = \frac{\theta}{m} \qquad i,j = 1, \dots, m$$
$$\Rightarrow \quad x_i' = x_i \left(\pi_i(x) - \pi(x) \right) \cdot (1 - \underline{\underline{\theta}})$$
$$+ \theta \left(\frac{1}{m} - \underline{\underline{x_i}} \right)$$