minule: $x' = F(x,t)$

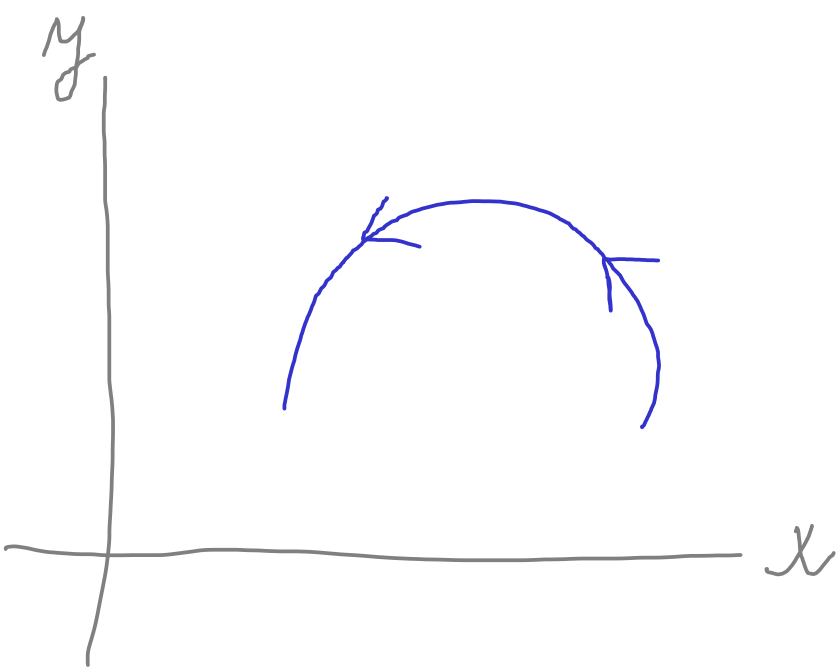
řešení: $t \mapsto x(t)$

$$x' \gtrless 0$$

$$x'' = \cdots \gtrless 0$$

nyní:
$$x' = f(x,y)$$
$$y' = g(x,y)$$

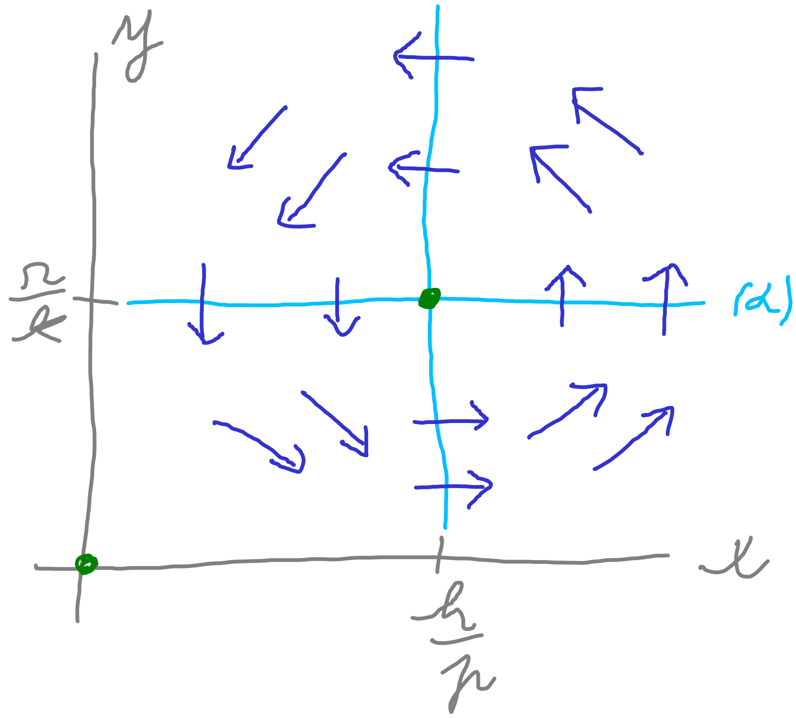
řešení: $t \mapsto (x(t), y(t))$



Př. Lotka-Volterra

$$x' = (r - ky)x$$
$$y' = (-h + px)y$$

$r, k, h, p > 0$
(konst.)



$\boxed{x' = 0}$: $x = 0$ nebo $r - ky = 0 \Leftrightarrow y = \frac{r}{k}$ (α)

$\boxed{y' = 0}$: $y = 0$ nebo $-h + px = 0 \Leftrightarrow x = \frac{h}{p}$ (β)

stac. body: $[0, 0]$, $\left[\frac{h}{p}, \frac{r}{k}\right]$

$\overset{??}{x' > 0}$ (pro $x > 0$) : $x' > 0$ (<) $\Leftrightarrow r - ky > 0$

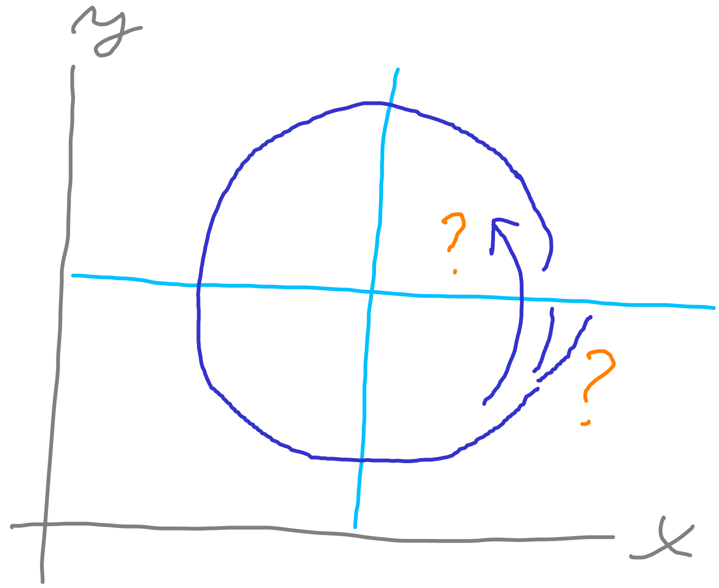
tj. $\boxed{y < \frac{r}{k}}$ (>)

$y' > 0$ (pro $y > 0$) : $y' > 0$ (<) $\Leftrightarrow -h + px > 0$

$\boxed{x > \frac{h}{p}}$ (<)

Př. Lotka-Volterra (pokračování)

$$x' = (r - ky)x$$
$$y' = (-h + px)y$$



globální chování?

TRIK: $$\frac{\frac{dy}{\cancel{dt}}}{\frac{dx}{\cancel{dt}}} = \frac{y'}{x'} = \frac{(-h+px)y}{(r-ky)x}$$

$$\frac{dy}{dx} = \frac{(-h+px)y}{(r-ky)x}, \quad y = y(x)$$

$$(r-ky)\frac{dy}{y} = (-h+px)\frac{dx}{x}$$

$$\left(\frac{r}{y} - k\right)dy = \left(-\frac{h}{x} + p\right)dx \qquad \int$$

$$r\ln y - ky = -h\ln x + px + C$$

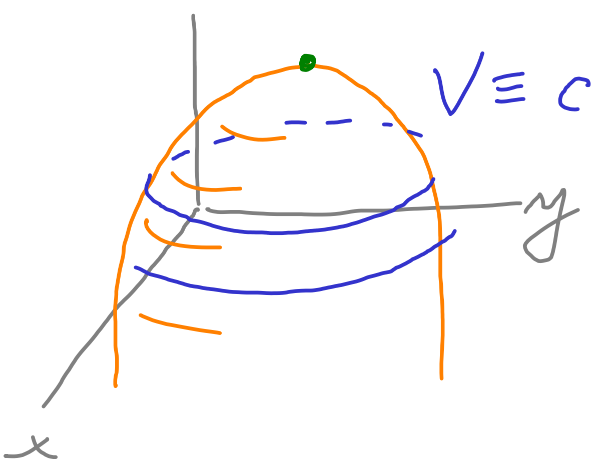
$$r\ln y - ky + h\ln x - px = C$$

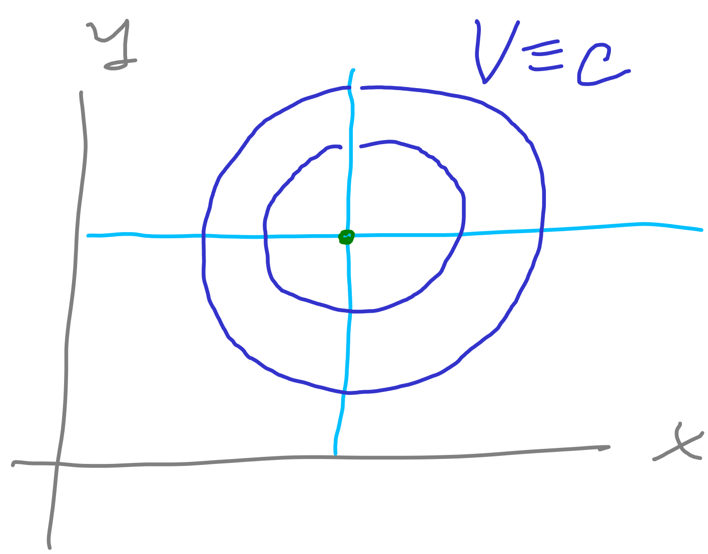
označ:

$$V(x,y) = \underbrace{r\ln y - ky}_{u(y)} + \underbrace{h\ln x - px}_{v(x)}$$

$\Rightarrow$ $V(x,y)$ … konstantní podél řešení
(tj. řešení = vrstevnice V)

Pozoruji: V ryze konkávní
($\Leftarrow V = u + v$, ryze konkávní)

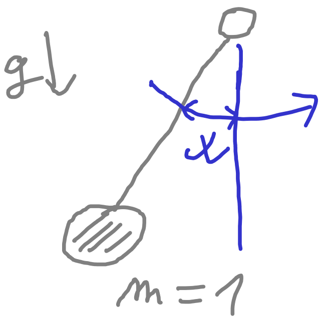


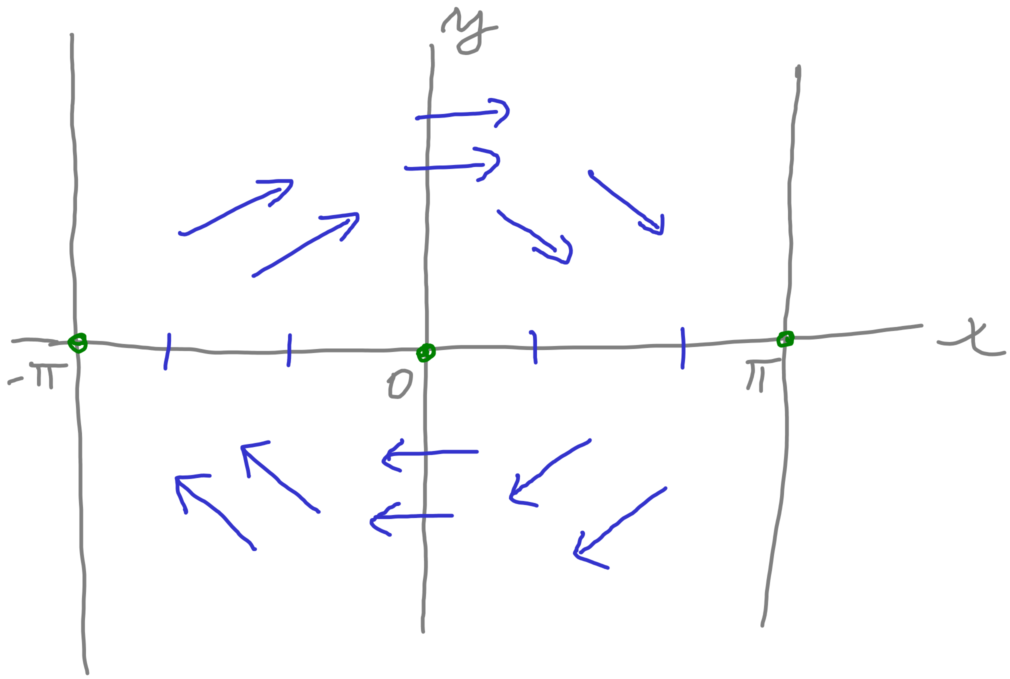


Terminologie: $V(x,y)$ je tzv. první integrál

Př. mat. kyvadlo:

$$x'' + \sin x = 0 \iff \begin{cases} x' = y \\ (x''=)\ y' = -\sin x \end{cases}$$





stac. body: $[k\pi, 0]$, $k \in \mathbb{Z}$

$x' > 0 \ (<) \iff y > 0 \ (<)$

$y' > 0 \ (<) \iff$ (pro $|x| \leq \pi$) $x \in (-\pi, 0)$ $\quad (x \in (0, \pi))$

TRIK → 1. integrál $\qquad x'' + \sin x = 0 \,/\, x'$

$x = x(t)$

$' = \frac{d}{dt}$

$$x'' \cdot x' + \sin x \cdot x' = 0$$

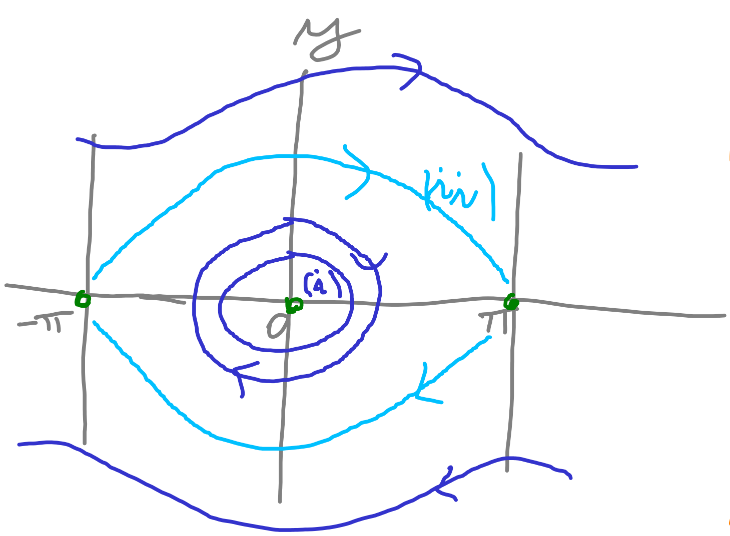
$$\left(\frac{(x')^2}{2}\right)' + (-\cos x)' = 0$$

$$\boxed{\frac{y^2}{2} - \cos x = c} \;/ +1$$

vzoreček: $2\sin^2\frac{x}{2} = 1 - \cos x$

⇒ $y^2 + 4\sin^2\frac{x}{2} = d$ (kde $d = 2(1+c)$)

$$\boxed{y = \pm\sqrt{\underline{d} - 4\sin^2\frac{x}{2}}}$$



0) $d \le 0$ … $\emptyset$

i) $d \in (0,4)$

(ii) $d = 4$ $\qquad y = \pm|\cos\frac{x}{2}|$

(iii) $d > 4$ $\qquad$ fyzikálne??

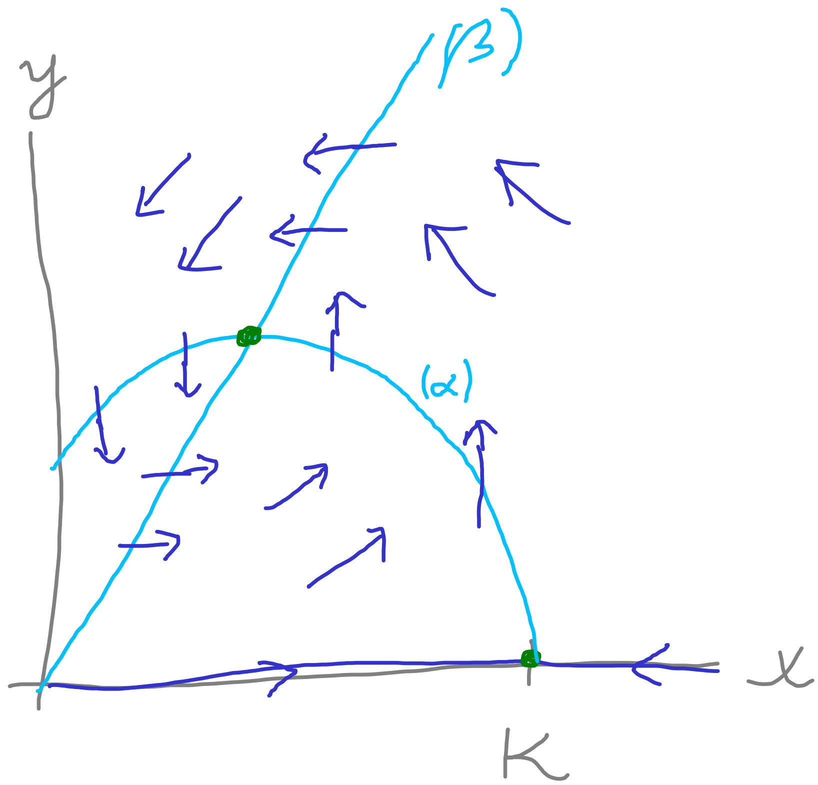
Př. Holling-Tanner ...

$$x' = \left(r\left(1-\frac{x}{K}\right) - \frac{m y}{A+x} \right) x$$

$$y' = s\left(1 - \frac{P y}{x}\right) y$$

.. ? 1. Q; tj

$x > 0,\ y \geq 0$



Pozn.: $y \equiv 0$ (2. rce o.k.)

$$x' = r\left(1-\frac{x}{K}\right) x$$

$x' = 0$: $\iff$ $y = \frac{r}{m}(A+x)\left(1-\frac{x}{K}\right)$ (α)

$y' = 0$ $\iff$ $y = \frac{x}{P}$ (β)

Dále: $x' > 0$ $(<)$ $\iff$ jsme pod/nad (α)

$y' > 0$ $(<)$ $\iff$ vlevo/vpravo od (β)