1) Consider the system

$$\begin{aligned} x' &= -x + 5y \\ y' &= -y \end{aligned}$$

Combine various information (matrix exponential, elementary analysis, first integral) to obtain accurate sketch of the solutions in the plane (x, y).
Is the origin stable or even asymptotically stable? Explain.

2) Consider the system

$$x' = y \tag{1}$$
$$y' = -2x^3 \tag{2}$$

i) Find a suitable first integral. What can be deduced about the stability of the origin?
ii) Consider the linearization at the origin. Is the linearized system stable?

3) Consider the repeated prisoner's dilemma

$$\begin{aligned} x' &= \frac{1}{4}x(1 - x - y)(5y - 4x - 4) \\ y' &= \frac{1}{4}y(1 - x - y)(5y - 4x - 1) \end{aligned}$$

where x is (always) altruistic, y is "tit-for-tat". Third strategy $z = 1 - x - y$ (always betraying) was excluded from the system. See the class for derivation. Use various tools (elementary analysis, linearization around stationary points, ...) to obtain accurate sketch of the solutions in the relevant domain

$$\Delta = \{x \geq 0,\ y \geq 0,\ x + y \leq 1\}$$

In particular, investigate the dynamics on the boundary of Δ.

Optionally, see the hints on page 2.

1) asymptotically stable - use Theorem P-6.
2i) dividing (y'/x'), or try $V = x^p + y^q$ *for suitable even integers. Stable, but not asymptically.*
2ii) linearization $=$ *remove* $-2x^3$ $\ldots$ *not stable anymore!*
3) factor $1/4$ *on the right-hand side can be removed (by time rescaling).*