

$$1) \quad \begin{aligned} x' &= 2x + y^2 - 1 \\ y' &= \sin x + 1 - y^2 \end{aligned}$$

$$E_1 = (0, -1) \quad A = \begin{pmatrix} 2 & -2 \\ 1 & 2 \end{pmatrix} \quad \lambda_{1,2} = 2 \pm i\sqrt{2}$$

\Rightarrow unstable

$$E_2 = (0, 1) \quad A = \begin{pmatrix} 2 & 2 \\ 1 & -2 \end{pmatrix} \quad \lambda_{1,2} = \pm\sqrt{6}$$

\Rightarrow unstable

$$2) \quad \begin{aligned} x' &= \ln(5 - 2x - 2y) \\ y' &= \exp(xy) - 1 \end{aligned}$$

$$E_1 = (0, 2) \quad A = \begin{pmatrix} -2 & -2 \\ 2 & 0 \end{pmatrix} \quad \lambda_{1,2} = -1 \pm i\sqrt{3}$$

\Rightarrow asymptotically stable

$$E_2 = (2, 0) \quad A = \begin{pmatrix} -2 & -2 \\ 0 & 2 \end{pmatrix} \quad \lambda_{1,2} = \pm 2$$

\Rightarrow unstable

$$3) \quad \begin{aligned} x' &= \left(1 - x - \frac{y}{x + 1/4}\right)x \\ y' &= \left(1 - \frac{4y}{3x}\right)y \end{aligned}$$

$$E = \left(\frac{1}{2}, \frac{3}{8}\right) \quad A = \begin{pmatrix} -7/6 & -2/3 \\ 3/4 & -1 \end{pmatrix} \quad \lambda_{1,2} = \frac{-7 \pm i\sqrt{47}}{12}$$

\Rightarrow asympt. stable

$$4) \quad x' = y \\ y'' = -\alpha y - \sin x$$

$$\triangleright F = \begin{pmatrix} 0, & 1 \\ -\cos x, & -\alpha \end{pmatrix}$$

$$E = (2k\pi, 0) \quad A = \begin{pmatrix} 0, & 1 \\ -1, & \alpha \end{pmatrix} \quad \lambda_{1,2} = \frac{-\alpha \pm \sqrt{D}}{2}$$

$$i) \quad \alpha^2 \geq 4 \Rightarrow D \geq 0, \text{ but}$$

$$D = \alpha^2 - 4$$

$$\sqrt{D} = \sqrt{\alpha^2 - 4} \leq \sqrt{\alpha^2} = |\alpha|$$

$$ii) \quad \alpha^2 < 4 \Rightarrow D < 0, \text{ i.e. } \sqrt{D} = \pm i\omega$$

In both cases: $\alpha > 0 \Rightarrow$ as. stable

$\alpha < 0 \Rightarrow$ unstable

$\alpha = 0 \dots \lambda = \pm i$ (??)

$$E = ((2k+1)\pi, 0) \quad A = \begin{pmatrix} 0, & 1 \\ 1, & \alpha \end{pmatrix}$$

$$\lambda = \frac{-\alpha \pm \sqrt{\alpha^2 + 4}}{2} \Rightarrow \text{unstable}$$

For any $\alpha \in \mathbb{R}$:

$$\sqrt{\alpha^2 + 4} > \sqrt{\alpha^2} = |\alpha|$$

$$\Rightarrow -\alpha + \sqrt{\alpha^2 + 4} > 0$$

$$5) \quad \begin{aligned} x' &= 1 - x^2 - y^2 \\ y' &= r^2 - x - y \\ r' &= r^2 - 1 \end{aligned}$$

$$\nabla F = \begin{pmatrix} -2x, -2y, 0 \\ -1, -1, 2r \\ 0, 0, 2r \end{pmatrix}$$

$$E_1 = (1, 0, -1) \quad \sigma(A) = \{-2, -1\} \Rightarrow \text{asympt. stable}$$

$$E_2 = (1, 0, 1) \quad \sigma(A) = \{-2, -1, 2\}$$

$$E_3 = (0, 1, -1) \quad \sigma(A) = \{-2, 1\}$$

$$E_4 = (0, 1, 1) \quad \sigma(A) = \{-2, 1, 2\}$$

} \Rightarrow unstable