ad1) Diagonal matrix $D = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \Rightarrow D^2 = \begin{pmatrix} a^2 & 0 \\ 0 & b^2 \end{pmatrix}$

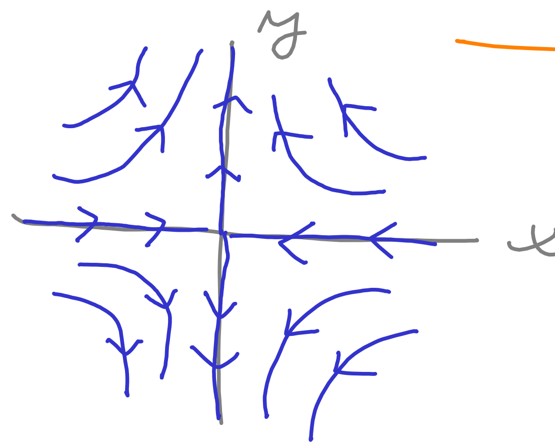
$$e^{tD} = \begin{pmatrix} e^{at} & 0 \\ 0 & e^{bt} \end{pmatrix}$$

... Here $a = 1$, $b = -1$ $\Rightarrow \boxed{\begin{pmatrix} e^{-t} & 0 \\ 0 & e^{t} \end{pmatrix}}$

first integral?

$$\frac{dy}{dx} = -\frac{y}{x}$$

$\Rightarrow \boxed{V = xy}$



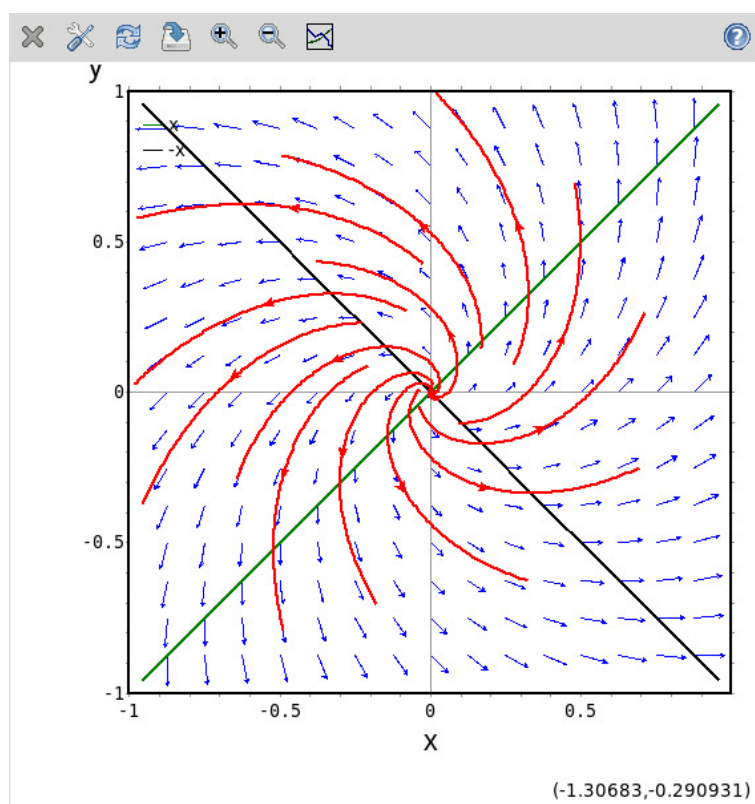
ad2) $A = I + B$, where $B = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

$$\Rightarrow e^{tA} = e^{tI} \cdot e^{tB} =$$

(See practicum, May 2)

$$= e^{t} \begin{pmatrix} \cos t, & -\sin t \\ \sin t, & \cos t \end{pmatrix}$$

spiralling out, counter clockwise



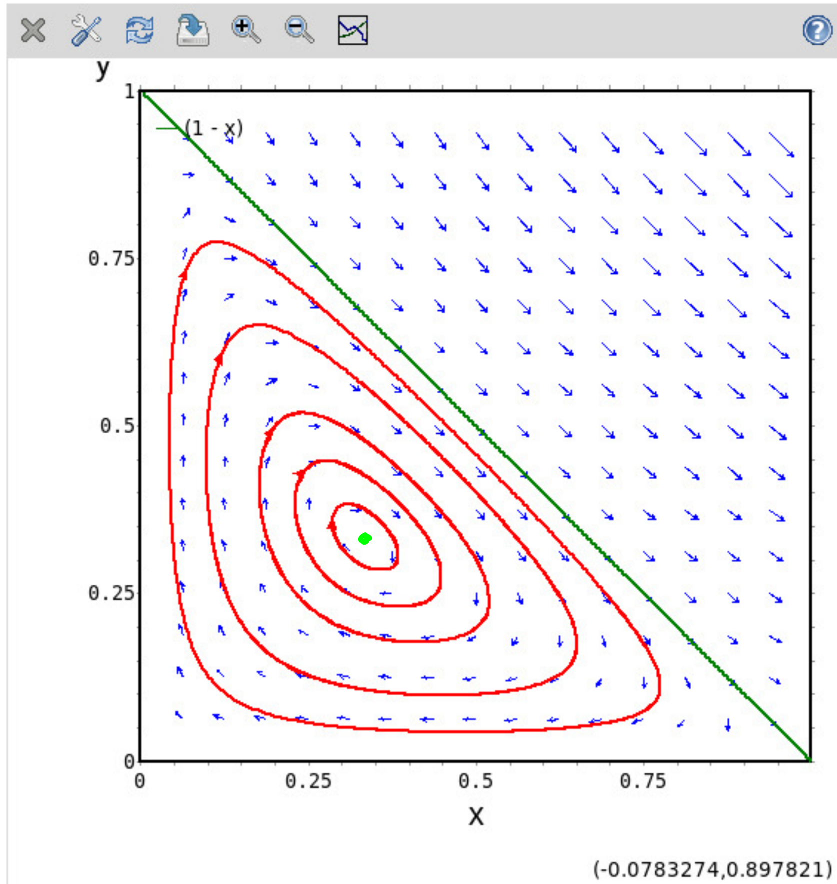
ad 3 $\begin{pmatrix} \pi_1(x) \\ \pi_2(x) \\ \pi_3(x) \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}}_{x} = \begin{pmatrix} x_2 - x_3 \\ x_3 - x_1 \\ x_1 - x_2 \end{pmatrix}$, $\pi(x) = 0$

(zero sum game!!)

(RD) $x_1' = x_1(x_2 - x_3)$
$x_2' = x_2(x_3 - x_1)$
$x_3' = x_3(x_2 - x_3)$

$\Updownarrow$ $x_1 = x, \; x_2 = y$
$x_3 = 1 - x - y$

$$\boxed{\begin{aligned} x' &= x(2y + x - 1) \\ y' &= y(1 - y - 2x) \end{aligned}}$$



$\exists$ periodic solutions around stationary point $\left(\frac{1}{3}, \frac{1}{3}\right)$
(stable, not asymptotically)

First integral: $V = x_1 x_2 x_3 = xy(1 - x - y)$

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ad 4 $A = \begin{pmatrix} 0, & 3, & -4 \\ 2, & 0, & -2 \\ 0, & 3, & 0 \end{pmatrix}$

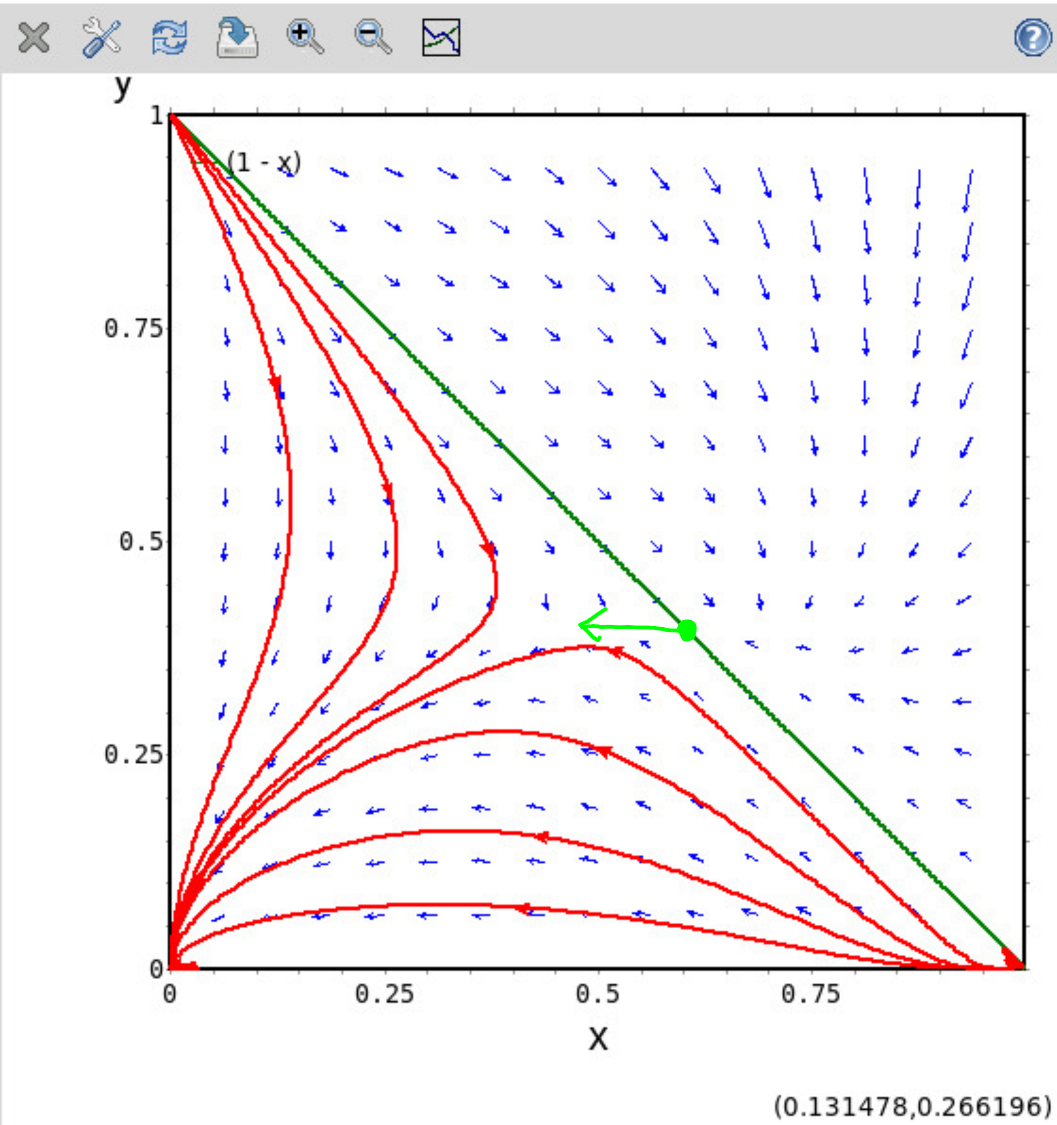
after normalization

$\underbrace{\begin{pmatrix} 6 & -12 & -6 \\ 0 & 0 & -8 \\ 6 & 0 & -6 \end{pmatrix}}_{\text{adj } A} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -12 \\ -8 \\ 0 \end{pmatrix}$

Thm 7.6 $\Rightarrow$ $\nexists$ stationary points inside

Set $x = x_1$ (hawk)
$y = x_2$ (bully)
$(x_3 = 1 - x - y$ (retaliator)$)$

$\Rightarrow$ $\boxed{\begin{aligned} x' &= x\left(y^2 - 8xy + 6y - 4x^2 + 8x - 4\right) \\ y' &= y\left(y^2 - 8xy + y - 4x^2 + 8x - 2\right) \end{aligned}}$



equilibria: 1) $(0,0)$ ... asymptotically stable,
(N.e. – pure R)

2) $\left(\frac{3}{5}, \frac{2}{5}\right)$ linearization matrix $A = \begin{pmatrix} \frac{3}{5} & \frac{8}{5} \\ 0 & -1 \end{pmatrix}$ $\left(\lambda_1 = \frac{3}{5}\right)$
$\lambda_2 = -1$

unstable $\left(\underline{v^{(1)} = (1,0)} \text{ ... unstable direction}\right)$