HW6. Consider the same system as in the previous homework, i.e.

$$x' = x(2 - x - y) \tag{1}$$
$$y' = y(x - 1) \tag{2}$$

For all the three equilibria, i.e. $(0,0)$, $(2,0)$ and $(1,1)$:

i) Find the linearization matrix and compute its spectrum, i.e. the eigenvalues.

ii) For any eigenvalue that is real, compute also the corresponding eigenvector(s).

Remark. Given the system $X' = F(X)$, in more detail

$$x' = F_1(x, y)$$
$$y' = F_2(x, y)$$

the linearization matrix is defined $A = \nabla \boldsymbol{F}(x_0, y_0)$, i.e. the gradient of the right-hand side, evaluated at a given equilibrium (x_0, y_0). The gradient is a 2×2 matrix, defined as

$$\nabla \boldsymbol{F} = \begin{pmatrix} \frac{\partial F_1}{\partial x} & \frac{\partial F_1}{\partial y} \\ \frac{\partial F_2}{\partial x} & \frac{\partial F_2}{\partial y} \end{pmatrix}$$