1. $x' - x = te^t \qquad$ i.f. $= e^{-t}$

$$(e^{-t}x)' = t$$

$$e^{-t}x = t^2/2 + C$$

$$x = e^t(t^2/2 + C) \qquad \boxed{t \in \mathbb{R}}$$

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2. $x' - \frac{x}{t} = t^2 e^t \qquad$ i.f. $= \frac{1}{t}$

$$\boxed{\begin{array}{l} t \in (-\infty, 0) \\ \text{or } (0, +\infty) \end{array}}$$

$$\left(\frac{x}{t}\right)' = te^t$$

$$x/t = C + \int te^t \, dt$$

$$\boxed{\text{integration by parts} \quad \int uv' = uv - \int u'v}$$

here: $u = t$, $v' = e^t$, $v = e^t$
$u' = 1$

$$\Rightarrow \int te^t = te^t - \int e^t = (t-1)e^t + C$$

(3) $x' - \frac{x}{t^2} = \frac{1}{t^3}$ i.f. $= e^{1/t}$

$$e^{1/t}x = C + \int \frac{1}{t^3} e^{1/t}$$

$\boxed{\text{Substitute:}}$

$u = \frac{1}{t}$

$du = -\frac{dt}{t^2}$

$$\leadsto -\int u e^u \, du = (1-u)e^u$$, see (2)

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(4) $x' + 2x = \cos t$, i.f. $= e^{2t}$

$$e^{2t}x = C + \int e^{2t} \cos t \, dt$$

<u>TRICK</u>: use by parts <u>twice</u> to get:

$$I = \int \underbrace{e^{2t}}_{u} \underbrace{\cos t}_{v'} = e^{2t} \sin t - 2 \int \underbrace{e^{2t}}_{u} \underbrace{\sin t}_{v'}$$

$$= e^{2t} \sin t - 2 \left\{ e^{2t}(-\cos t) + 2 \underbrace{\int e^{2t} \cos t}_{I \text{ again!!}} \right\}$$

$$\Rightarrow I = \frac{1}{5} e^{2t} (2\cos t + \sin t)$$