① $F(a)=\int_1^{\infty} \underbrace{\frac{x^2-1}{x^a \ln x}}_{f(a,x)}\, dx$;

[8b]

$1+$: $\frac{1}{x^a}\cdot\underbrace{\frac{x-1}{\ln x}}_{\to 1}\cdot(x+1)$: $\int_1^{1+\delta} < \infty$ pro $\forall a$.

∞ : $f(a,x) \sim \frac{1}{\ln x}\cdot\frac{1}{x^{a-2}}$; konv. pro $a-2>1$

$\boxed{a>3}$.

2

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derivování dle parametru: $f(a,\cdot)$ měřitelné $\in$ spojité

$$\frac{\partial f}{\partial a} = \frac{x^2-1}{\ln x}\cdot\frac{\partial}{\partial a}\exp(-a\ln x)$$

$$= \frac{1-x^2}{x^a}$$ ... konečné pro $\forall x\in(1,\infty)$.

1

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majoranta: $J=(3+\eta,\infty)$; $\eta>0$ pevné:

$$\sup_{a\in J}\left|\frac{\partial f}{\partial a}\right| = \frac{x^2-1}{x^{3+\eta}} \le \frac{1}{x^{1+\eta}} \in L^1(1,\infty).$$

2

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konvergence: $a_0=4$ (BÚNO $\eta<1$)

$$f(4,x)=\frac{x^2-1}{x^4\ln x}\in L^1(1,\infty)$$ ... viz výše.

1

$$F'(a) = \int_1^\infty \frac{1-x^2}{x^a} = \int_1^\infty \left(\frac{1}{x^a}-\frac{1}{x^{a-2}}\right)dx = \frac{1}{a-1}-\frac{1}{a-3} ;$$

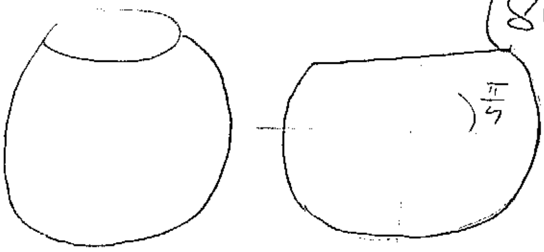
$$\Rightarrow F(a)=\ln\frac{a-1}{a-3}+C ;$$

$$\lim_{a\to\infty}F(a)=0 \Rightarrow C=0.$$

2

(2) $\int_{\partial P} \underset{\sim}{F}\cdot d\underset{\sim}{s} = \int_{P} \mathrm{rot}\,\underset{\sim}{F}\cdot d\underset{\sim}{S}$. (8b)

$F=(1,z^2,z^3)$



1

(LS): $\varphi:\ x=\frac{1}{\sqrt2}\cos t$, $\quad \varphi'(t)=\frac{1}{\sqrt2}(-\sin t,\cos t,0)$

$y=\frac{1}{\sqrt2}\sin t$

$z=\frac{1}{\sqrt2} \qquad t\in[0,2\pi]$. 1

$$\int_0^{2\pi}\left(1,\frac12,\frac{1}{2\sqrt2}\right)\cdot(-\sin t,\cos t,0)\cdot\frac{1}{\sqrt2}\,dt=\frac{1}{\sqrt2}\int_0^{2\pi}\left(-\sin t+\frac12\cos t\right)dt=0.$$ 1

(PS): $\mathrm{rot}\,\underset{\sim}{F}=\begin{matrix}(\partial_x,\partial_y,\partial_z)\\(1,z^2,z^3)\end{matrix}=\left(-\overset{2z}{\cancel{x}},0,0\right)$.

$\psi:\ \left.\begin{matrix} x=\cos u\sin v & u\in(0,2\pi)\\ y=\sin u\sin v & v\in\left(\frac{\pi}{4},\pi\right).\\ z=\cos v & \end{matrix}\right\}\Omega$ 2

$\partial_u\psi=(-\sin u\sin v,\ \cos u\sin v,\ 0)$

$\partial_v\psi=(\cos u\cos v,\ \sin u\cos v,\ -\sin v)$

$\partial_u\psi\times\partial_v\psi=(-\cos u\sin^2 v,\ \dots)$

$$(PS)=2\int_\Omega(+\cos u\sin^2 v)\overset{\cos v}{\vee}\,du\,dv=\left(\int_0^{2\pi}\cos u\,du\right)\int_{\frac{\pi}{4}}^{\pi}\sin^2 v\overset{\cos v}{\vee}=0.$$ 2

1

? shoda orientací: ...

(3) $\Phi(y) = \int_1^4 (y')^2 + \frac{3}{4x^2} y^2 \, dx$; $y(1) = 1$, $y(4) = 8$ [8b]

$f = r^2 + \frac{3}{4x^2} y^2$;

$f_r = 2r$; $f_y = \frac{3}{2x^2} y$;

(E.L.) $-(2y')' + \frac{3}{2x^2} y = 0$

$x^2 y'' - \frac{3}{4} y = 0$; 3

$\boxed{y = x^\lambda}$ char. rce: $\lambda(\lambda-1) - \frac{3}{4} = 0$

$\lambda^2 - \lambda - \frac{3}{4} = 0$

$(\lambda - \frac{3}{2})(\lambda + \frac{1}{2}) = 0$

F.S.: $x^{3/2}, x^{-\frac{1}{2}}$;

$y(x) = Ax^{3/2} + Bx^{-\frac{1}{2}}$;

$x=1$: $A + B = 1$ $\Rightarrow A = 1$ $\boxed{y = x^{3/2}}$ 2

$x=4$: $8A + \frac{1}{2}B = 8$ $B = 0$

Jacobi: $f_{rr} = 2$;

$f_{yr} = 0$

$f_{yy} = \frac{3}{2x^2}$

(J): $2u'' - \frac{3}{2x^2} u = 0$

$x^2 u'' - \frac{3}{4} u = 0$

$u(x) = \alpha x^{3/2} + \beta x^{-\frac{1}{2}}$

$u(1) = 0$: $\alpha = -\beta$

$u = \alpha(x^{3/2} - x^{-\frac{1}{2}}) \ldots$

monotónní funkce: nemá konj. bod.

$\rightarrow$ lokální minimum.

(4) $f(x) = \sin 2x$; $x \in (0, \pi)$ (86)

sudé ; 2π - periodické.  1

$b_k = 0$;

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) = \frac{2}{\pi}\int_0^{\pi} \sin 2x \, dx = \frac{2}{\pi}\left[-\frac{\cos 2x}{2}\right]_0^{\pi} = 0. \qquad 1$$

$$\frac{\pi}{2} a_k = \int_0^{\pi} \sin 2x \cos kx \, dx = \int_0^{\pi} \frac{1}{2}\left[\sin(k+2)x - \sin(k-2)x\right] dx ;$$

$k \geq 1$ vzorec: $\sin a \cos b = \frac{1}{2}\left[\sin(a+b) + \sin(a-b)\right]$

$$\int_0^{\pi} \sin(k+2)x \, dx = \left[-\frac{\cos(k+2)x}{k+2}\right]_0^{\pi} = \frac{1}{k+2}\left(1 - \cos(k\pi + 2\pi)\right) = \frac{1-(-1)^k}{k+2}.$$

$$\int_0^{\pi} \sin(k-2)x \, dx = \begin{cases} 0 & k = 2 \\ \dfrac{1-(-1)^k}{k-2} ; & k = 1, 3, \dots \end{cases} \qquad 2$$

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celkem: $a_{2l} = 0$;

$$l \geq 0: \quad a_{2l+1} = \frac{2}{\pi}\left(\frac{1}{2l+3} - \frac{1}{2l-1}\right) = -\frac{8}{\pi} \cdot \frac{1}{(2l+3)(2l-1)} \qquad 1$$

$$F_f(x) = -\frac{8}{\pi}\sum_{l=0}^{\infty} \frac{\cos(2l+1)x}{(2l+3)(2l-1)} ; \qquad 1$$

$f(x)$ spojitá, po částech C^1 : $F_f(x) = f(x)$ pro $\forall x \in \mathbb{R}$. 2